Applied mathematical modelling at the University of Limerick: A new era in Irish mathematics

"*Too few people recognize that the high technology so celebrated today is essentially a mathematical technology*". Edward E. David, (President of Edward E. David, Inc., advisors to industry, government and academia on technology, research and innovation, ex-President of Exxon R & D, ex-Executive Director of Bell Telephone Laboratories.)

- What are the moving black lines evident in a settling pint of stout and how are these linked to the water waves on an inclined rough road (roll-waves) and the flow of traffic on a motorway? These are both manifestations of a type of periodic wave called a roll-wave. Roll-waves in river flows were originally investigated by Dressler (1949). In a settling pint of stout, the mixture consists of nitrogen bubbles and (viscous) liquid. The waves are embedded in a circulatory convective flow which is descending near the wall of the glass. (See Figure **??**). In traffic flow on a motorway, even though the basic traffic flow is all in one direction, when viewed from a helicopter above the motorway, one can observe waves of car concentration moving in either direction along the line of cars. For example, a traffic jam is a wave moving in the opposite direction to the basic flow of cars.

- How does a widget in a can of stout work? Open question.

- Can blown fuses in integrated circuits spontaneously heal? Open question.

- How can one reliably etch 30 nm. photo-resist lines on a silicon substrate to within a certain tolerance?

  This is a complicated problem which requires solution of a mathematical model of the reaction diffusion processes involved in etching.

- How can one best process DNA in a microfluidic cell?

  This requires solution of a mathematical problem involving heat, mass and momentum transfer in a low Reynolds number (i.e., very viscous) two phase flow (i.e., the fluid phase consists if two separate phases).

- How quickly does spilt fuel on an airport runway migrate through the underlying soil towards the water table and how is this mathematically connected with the problem of cooking hamburgers without burning them?

  Liquids drain under gravity through aquifers at different rates, depending of course on the make-up of the aquifer. Typically the velocities are of the order of 1 cm per day. If fuel is spilt into an initially dry (unsaturated) aquifer, one can envisage a plume of liquid draining down under gravity, and the confines of the moving wet area defines by a *moving boundary.* Similarly, if a frozen hamburger is placed on a pan and heated from below, the burger thaws from beneath, with the thawing *boundary moving* upwards through the burger in much the same way that the ice-water boundary in a block of melting ice appears to move. The hamburger also has an evaporation front, delineating the cooked and uncooked regions.

- What is the best way to monitor foetal distress from a consideration of foetal heart-rate?

  Most mathematicians would now favour so-called phase-space embedding techniques over traditional statistical techniques.

- How can we predict the amount of antigen present in a sample based on the current response measured by an electrode?

  This requires solution of a complicated advection/recation/diffusion mathematical model.

- How is the pricing of options connected to the coating of fluorescent light-bulbs or the dripping of paint from a ceiling under gravity?

  The connection is a bit tenuous but the celebrated Black-Scholes equation of economics (which predicts option prices) is technically a backward diffusion equation; backward diffusion is also the effect which causes droplets to form and eventually to fall from a freshly painted ceiling.

- Why do craters tend to appear in a drying paint layer and why is this connected to the "tears of strong wine"?

  Both of these effects are associated with surface tension variations caused by evaporation from the respective mixtures. Pour a half glass of cognac and a glass of water into identical brandy glasses, place on a table and allow to stand for a minute or so. Without moving either glass, observe the part of the glass just above the upper surface of the liquid in each case. Above the water, you will find the glass more or less permanently dry but above the meniscus of the liquid you will observe droplets of liquid continually draining back down into the cognac. These are the visible manifestation of a more or less continuous flow of a very thin film of liquid **up** the glass against gravity. The flow is driven by evaporation and surface tension gradients caused by the fact that cognac is effectively a mixture of water and methanol and the latter can change the surface tension of the water. The wine has to be "strong" to ensure that the gradients in surface tension are sufficiently strong.

These are the sorts of questions which the applied mathematical modeller asks; if such questions interest you, you should read on.

Supported by Science Foundation Ireland (Mathematics Initiative) funding of Eur 4.43m we are establishing an Irish applied mathematical modelling group (MACSI: Mathematics Applications Consortium for Science and Industry), centred at the University of Limerick, which will develop a coherent strategy for the solution of problems which arise in science, engineering and industry in Ireland and will involve collaboration with industries such as Analog Devices Limerick, Dell Computers, Diageo, Waterford Crystal, Boston Scientific, Kostal, Transitions Optical. This is the largest single award ever made to mathematics in this country and signifies a recognition of the academic and strategic importance of applied mathematics in a growing economy.

What is applied mathematical modelling? In practice, the terms "industrial mathematics", "applied mathematics" and "mathematical modelling" are often used as near synonyms to make the distinction with pure mathematics whose central tenet is formal proof and which is not generally concerned with real problems arising outside of mathematics. An applied mathematician is a kind of lapsed pure mathematician in the sense that he/she would like to prove every result formally but is sometimes unable to do so and must make intuitive leaps in the search for understanding. Otherwise the flow of air over airplane wings would not be understood.

It is important to understand that "applied mathematics" encompasses a much broader field than the present Leaving Certificate syllabus. In the EU commissioned project "Tuning educational structures in Europe" (in Newsletter of the European Mathematical Society, September 2002, pp.26–28), the Mathematics Subject Area group identified the ability to mathematically model a situation as a key skill which every mathematics graduate should acquire. Classical applied mathematics is associated with names such as Archimedes, Newton, G.I Taylor, Stokes, Reynolds, Kelvin all of whom regularly used mathematics to understand phenomena in the physical world, essentially operating as mathematical modellers. Of course, in their times there was no serious industry requiring mathematical modelling on a large scale but if there had been there is little doubt that they would have been involved in it. It was only late in the nineteenth century that pure mathematicians, who do not usually seek inspiration from the world in which they live, emerged.

In fact applied mathematical modelling does not specify a novel kind of mathematics, but rather a philosophy of asking how things work and where emphasis is placed on the application of mathematics in non-mathematical disciplines e.g. in finance, economics, biology, physics, chemistry or industry (which can be considered a complex world in itself often embracing all of the above disciplines). A problem or phenomenon of some sort occurs outside mathematics and mathematics is used to explain, to understand or to improve it. The emphasis then is not on the mathematics itself, but on the use of mathematics to understand a phenomenon in a non-mathematical world. At its simplest, a farmer whose field forms a right angled triangle, whose two shorter lengths are 30 and 40 metres, can use a simple mathematical model to deduce that the longest side is 50 metres long without having to measure this side. Mathematical modellers perceive themselves as being scientists as well as mathematicians and are interested in other disciplines apart from mathematics. Without this philosophy, most modern technology would not exist: airplanes would not fly, man would not have reached the moon, there would be no scientific weather forecasts. There are even mathematical models for the dynamics of marriage!

In addition, mathematical modelling encourages structured thinking and in many cases leads experimentalists to an improved experimental design. Indeed, in modern industry, mathematical modelling often replaces experiment, including animal experiment.

Applied mathematical modelling is apparent in most areas of modern life and this is corroborated by the existence of mathematical modelling groups at Cambridge, Oxford and throughout Europe in the context of the European Consortium for Mathematics in Industry of which the University of Limerick was a founder member.

In practice, novelty has always followed the application of mathematics to real world problems. An obvious example of this is Newton and the development of the calculus. This arose from his practical investigations in mechanics and geometry and the need to measure areas under curves. Similarly, differential geometry was invented in the context of making maps and surveying a region of the earth. Fourier and the desire to understand heat conduction problems led to the development of modern Fourier theory. The study of the way that fluids flow has led to significant scientific advances in many unassociated areas; our current understanding of chaos and turbulence arose from Lorentz's investigation of a simplified mathematical model for the flow of air in the atmosphere. As is typical in applied mathematics, the scientific understanding arising from a mathematical model has turned out to be a common (universal) feature of many apparently unassociated phenomena.

The UL undergraduate mathematical sciences degree places a strong emphasis on mathematical modelling. As part of this new SFI funded initiative, with the support of national and international experts, we will be establishing a Masters course in industrial mathematical modelling and scientific computation which will become part of the Fourth Level (Ph.D) programme in the country and should be an aspiration for an mathematicians with an interest in mathematical modelling.

Stephen O'Brien, Professor of Applied Mathematics, Department of Mathematics, University of Limerick (UL). http://www.staff.ul.ie/obriens/index.html http://www.macsi.ul.ie

Figure 1: Spot the difference: roll-waves on a lane-way in Craggaunowen, Co. Clare and waves in a pint of stout.