§ 1.6 , p 35

#2 $y'' - y = \sin^2 x$

1) $y_h(x) = c_1 \underbrace{e^{x}}_{y_1(x)} + c_2 \underbrace{e^{-x}}_{y_2(x)}$

2) $y_p(x) = u\, e^{x} + v\, e^{-x}$

...

$$\begin{cases} u' e^{x} + v' e^{-x} = 0 \\ u' e^{x} - v' e^{-x} = \sin^2 x \end{cases}$$

$$\Longrightarrow \begin{cases} u' = \frac{1}{2} e^{-x} \sin^2 x \\ v' = -\frac{1}{2} e^{x} \sin^2 x \end{cases}$$

Hint : use $\sin^2 x = \dfrac{1-\cos(2x)}{2}$

to integrate

...

$$u(x) = \int \frac{e^{-x}}{4} dx - \int \frac{e^{-x}\cos(2x)}{4} dx$$

$$= -\frac{e^{-x}}{4} + e^{-x}\left(\frac{\cos(2x)}{20} + \frac{\sin(2x)}{10}\right)$$

$$v(x) = -\int \frac{e^{x}}{4} dx + \int \frac{e^{x}\cos(2x)}{4} dx$$

$$= -\frac{e^{x}}{4} + e^{x}\left(\frac{\cos(2x)}{20} + \frac{\sin(2x)}{10}\right)$$

$$\Longrightarrow \quad y_p(x) = \underbrace{u}\, e^{x} + \underbrace{v}\, e^{-x} = \dots$$

use our findings

$$= -\frac{1}{2} + \frac{1}{10}\cos(2x)$$

<u>Answer</u>:
$$\boxed{y(x) = -\frac{1}{2} + \frac{1}{10}\cos(2x) + C_1 e^{x} + C_2 e^{-x}}$$

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#3 $\quad y'' - 4y' + 4y = \dfrac{e^{2x}}{1+x}$

1) $y_h(x) = C_1 e^{2x} + C_2 \cdot x e^{2x}$

2) $y_p(x) = u \underbrace{\boxed{e^{2x}}}_{y_1} + v \cdot \underbrace{\boxed{x e^{2x}}}_{y_2}$

$$\begin{cases} u' y_1 + v' y_2 = 0 \\ u' y_1' + v' y_2' = Rx \end{cases} \Longrightarrow$$

$$\begin{cases} u' e^{2x} + v' \cdot x e^{2x} = 0 \\ u' \cdot 2e^{2x} + v'(2x e^{2x} + e^{2x}) = \dfrac{e^{2x}}{1+x} \end{cases}$$

$$\begin{cases} u' = -x v' \\ \boxed{v' = \dfrac{1}{1+x}} \end{cases} \Longrightarrow \boxed{\begin{aligned} u' &= -\frac{x}{1+x} \\ &= \frac{1}{1+x} - 1 \end{aligned}}$$

$$u = \int \left(\frac{1}{1+x} - 1 \right) dx = \ln(1+x) - x$$

$$v = \int \frac{1}{1+x}\, dx = \ln(1+x)$$

$$\Rightarrow y_p(x) = \underbrace{(\ln(1+x) - x)}_{u} e^{2x} + \underbrace{\ln(1+x)}_{v} \cdot x e^{2x}$$

$\Longrightarrow$

$$y(x) = y_p(x) + y_h(x)$$

$$\boxed{\begin{aligned} y(x) &= \left[\ln(1+x) - x + x \cdot \ln(1+x)\right] e^{2x} \\ &\quad + C_1 e^{2x} + C_2 \cdot x e^{2x} \end{aligned}}$$