Chapter 4

Numerical methods

Exercise 1

(i) Find the slope of the tangent to the curve $y = x^2 - 3x + 2$ when $x = 1$.

(ii) Find the equation of the tangent at this point.

Exercise 2

Using mostly derivative information, sketch the graph of

$$f(x) = x^3 - 4x^2 + x + 6$$

Note that we need information about the function such as critical points, roots... To find the roots, we remark that $x^3-4x^2+x+6 = (x+1)(x-2)(x-3)$.

Exercise 3

(i) Consider the same $f(x)$ as in the previous question. Use the right initial guess x_0 to estimate the root at $x = -1$, $x = 2$, and $x = 3$.

(ii) Are there any choices of x_0 for which the method fails to find a root?

Exercise 4

(i) Using derivative information, sketch the graph of the function

$$f : \mathbb{R} \longmapsto \mathbb{R}, \qquad f(x) = x^3 - 5x^2 + 8x - 3.$$

(find critical points etx...)

(ii) Use Newton's method to approximate the root(s) of this function. Note, we can sue the graph in (i) to determine approximate value(s) of our initial guess(es) x_0.

(iii) Sketch another graph of the function incorporating the root(s) obtained in (ii).

Exercise 5

When using the Newton-Raphson method of root finding, suppose our initial guess x_0 is lucky and x_0 is a root of f, which means $f(x_0) = 0$. What happens to the next approximation x_1 and later approximations?