

Chapter 6

Linear algebra

Exercise 1

Consider the following matrices.

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 8 & 2 \end{pmatrix}, \quad D = (1 \ 4),$$

$$E = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad F = \begin{pmatrix} 7 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 5 & 4 \end{pmatrix}, \quad G = \begin{pmatrix} 3 & 4 \\ 9 & 5 \end{pmatrix}.$$

Calculate the following sums and products if possible:

1. $3C$
2. $2A - G$
3. $E + 3F$
4. $C + A$
5. AG
6. AC
7. CA
8. AB
9. BD
10. EF
11. FE

Exercise 2

Find the transpose of all matrices in the previous exercise.

Exercise 3

Show that

1. $(AG)^T = G^T A^T$
2. $(AC)^T = C^T A^T$
3. $(AB)^T = B^T A^T$
4. $(EF)^T = F^T E^T$

Exercise 4

1. Sketch the following three vectors.

$$u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad v = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} 4 \\ -2 \end{pmatrix}.$$

2. Find the length of each of these vectors.
3. Find the angle between each pair of vectors.

Exercise 5

Find the length and midpoint of the line segments:

1. with end points $(5, -7)$ and $(8, -11)$.
2. whose endpoints are defined by the vectors $(-1//12)$ and $(11//7)$.

Exercise 6

1. Translate the line segment with endpoints $(5, -7)$, $(8, -11)$ three units up and one unit to the left.
2. Find the length of this new line segment. Is it the same as in exercise 2, (a)?

Exercise 7

Rotate the line segment with endpoints $(0, 0)$, $(3, 3)$ anti-clockwise by $\pi/4$ radians (45°) about the origin.

Exercise 8

Rotate the line segment with endpoints $(2, 2)$, $(3, 3)$ anti-clockwise about the endpoint $(2, 2)$ by $\pi/2$ radians. (Note: first you must translate the line segment so the endpoint $(2, 2)$ is at the origin, then perform the rotation, and then reverse the translation.)

Exercise 9

Consider the line segment with endpoints $(2, 2)$, $(4, 6)$.

1. Find its length.
2. Rotate the line segment $\pi/2$ radians anti-clockwise about its midpoint.
3. Find the length of this new line segments. Is it the same?