Chapter 1

Graph theory

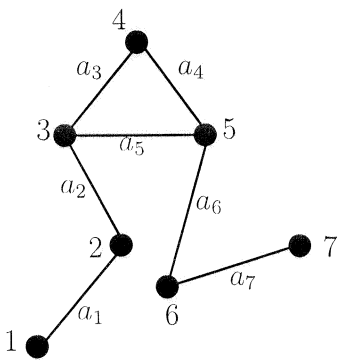
Exercise 1

Sketch a graph having nodes $\{1, 2, 3, 4, 5\}$, arcs $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ and with rule:

$$a_1 = 3 - 4, \quad a_2 = 1 - 2, \quad a_3 = 3 - 4, \quad a_4 = 1 - 1$$
$$a_5 = 2 - 3, \quad a_6 = 1 - 5, \quad a_7 = 5 - 5.$$

Exercise 2

Consider the follwing graph



(i) Is it simple?

(ii) Is it complete?

(iii) Is it connected?

(iv) Find a path from node 1 to node 6.

(v) Are there any cycles in the graph?

(vi) Is it possible to remove an arc so the resulting graph is a tree?

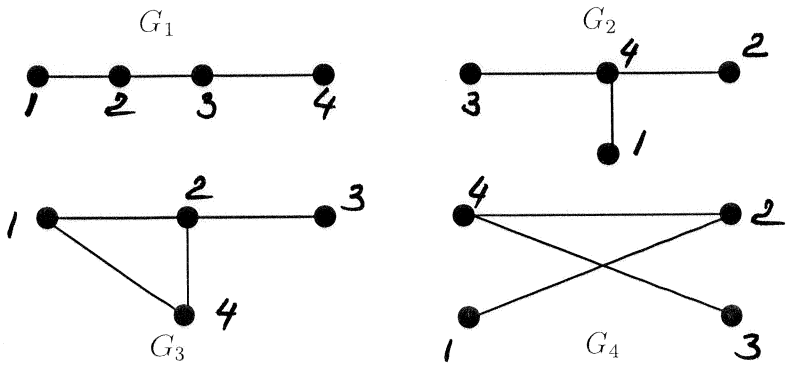
(vii) Is it possible to remove an arc so the resulting graph is not connected?

Exercise 3

Find a connected graph that is not complete.

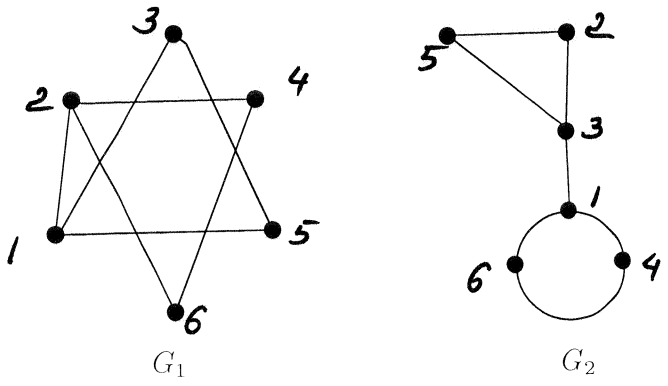
Exercise 4

Are any of the following graphs isomorphic to each other?



Exercise 5

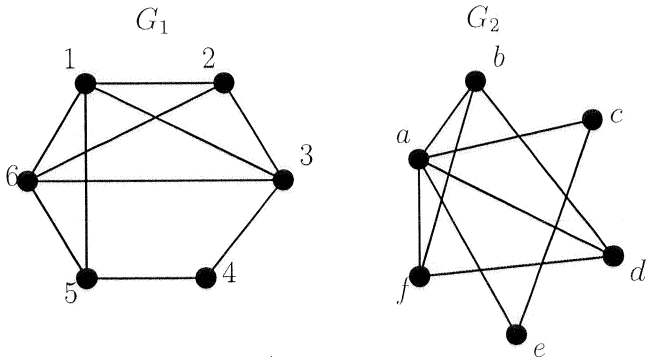
Construct an isomorphism between the following graphs:



Exercise 6

(i) Draw the graphs : K_4, $K_{1,3}$, $K_{3,4}$.

(ii) Redraw the following as planar graphs and verify Euler's formula for each of them.



(iii) How many edges must be drawn to obtain a connected planar graph with 7 nodes and 7 regions?

Exercise 7

(i) Draw all non-isomorphic trees with 5 nodes.

(ii) A football tournament is played with 9 teams. We denote these teams by T_i, $i = 1, \ldots 9$. We design the tournament so that in order for team T_i for $i = 1, \ldots 8$ to win, they must play i games. Model such a situation with a tree and determine how many games must the team T_9 play in order to win the tournament.

(iii) How many leaves are in a binary tree with 5 interior nodes?

(iv) Draw a tree to represent the following algebraic expressions:

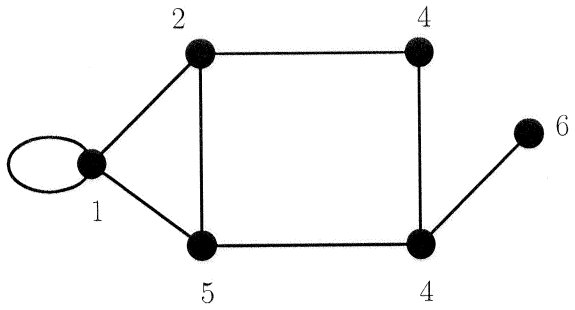
a) $(2 + x)^2 * ((2 - y)/(7 + x))$.

b) $((3 + z) * ((x - y) + 4)) - x^2$

Exercise 8

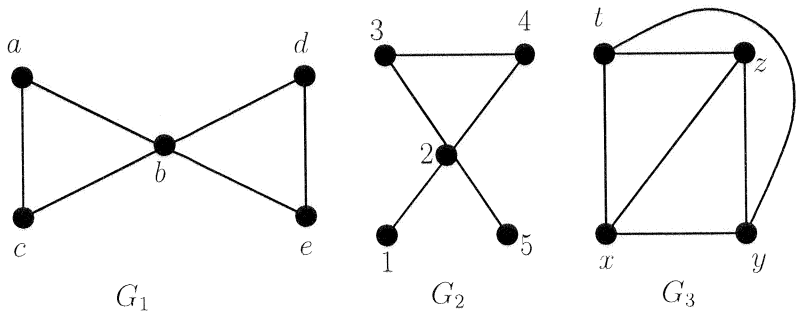
(i) Construct the adjacency matrix for the following graph.

(ii) Suppose we consider a simple graph. What can we say about its adjacency matrix?



Exercise 9

Find an Euler Circuit or an Euler path in each of the graphs below or say that neither exist.

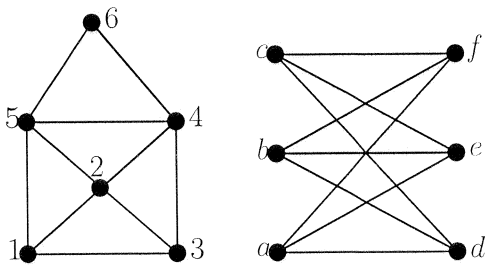


G_1 G_2 G_3

Exercise 10

(i) Show the complete graph K_4 is Hamiltonian.

(ii) Is there a Hamiltonian circuit in the following graphs, if not do they have a Hamiltonian path?



G_1 $K_{3,3}$