UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4402

SEMESTER: Autumn 2014

MODULE TITLE: Computer Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Prof. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Answer Questions 1, 2, 3 and 4.**

To obtain maximum marks you must show all your work clearly and in detail.

Some useful formulae are given on the final page. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.

Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted.

1 Answer part (a) and *one* of parts (b) and (c). **20%**

. . . .

(a) Consider the graph $G = (V, E)$ with the vertex set $V = \{0, 1, 2, 3, 4, 5\}$ and the edge set

$E = \{0-3,\ 0-5,\ 1-2,\ 1-3,\ 1-4,\ 2-3,\ 2-4,\ 2-5,\ 4-5\}.$ 8%

(i) Sketch this graph.

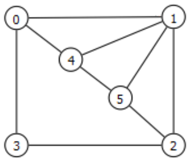
(ii) How many nodes and edges are in this graph?

(iii) Redraw this graph as a planar graph. Show that it satisfies Euler's formula for planar graphs.

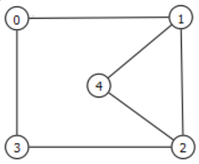
. . . .

(b) Which of the three graphs G_1, G_2, and G_3 below is isomorphic to the graph G from part (a)? In each case, justify your answer, i.e. either establish an isomorphism, or provide an argument that none exists. 12%

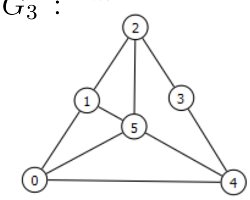
G_1 :



G_2 :

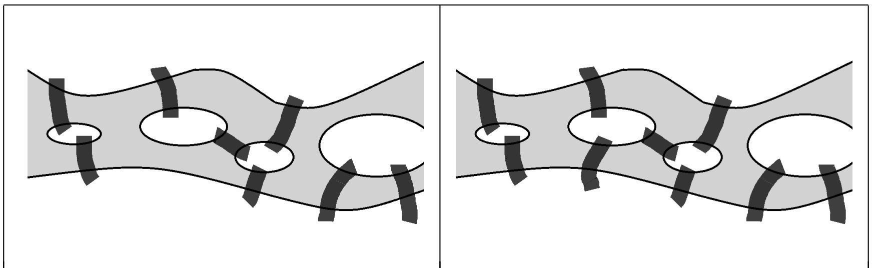


G_3 :



. . . .

(c) Consider the two variations of the Königsberg bridge problem: 12%



Variation A Variation B

The river divides the city into 2 banks and 4 islands, flowing under 8 bridges (Variation A) or 9 bridges (Variation B).

(i) Is it possible to take a walk that crosses each bridge exactly once?

(ii) Is it possible to take a walk that crosses each bridge exactly once and finishes where it starts?

Hint: Reformulate each problem using a graph. Check whether the graph is Eulerian and/or traversable.

2 Answer part (a) and *one* of parts (b) and (c). **20%**

. . . .

(a) Evaluate the matrix $A^2 - 2A + B$, and hence the determinant $\det(A^2 - 2A + B)$ for the matrices 8%

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -3 & 2 \\ 1 & 0 & 0 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 & 9 & -5 \\ 1 & 1 & -1 & 2 \\ 2 & 2 & -2 & 6 \\ 1 & 0 & 2 & 1 \end{bmatrix}.$$

. . . .

(b) Consider the points $A = (-1, 2, -3)$, $B = (4, -1, 2)$, $C = (3, 4, -1)$ in the coordinate space. 12%

(i) Find the cross product $\overrightarrow{AB} \times \overrightarrow{AC}$. Use it, to find the area of the triangle $\triangle ABC$.

(ii) Using the dot product, check whether $\triangle ABC$ is a right-angled triangle. Justify your answer.

. . . .

(c) Consider the points $A = (3, -1)$, $B = (4, 2)$ and $C = (1, 2)$ on the coordinate plane. 12%

(i) Describe the triangle $\triangle A'B'C'$ obtained by rotating the triangle $\triangle ABC$ by the angle $\frac{\pi}{3} = 60^o$ in the clockwise direction about the point C.

(ii) Evaluate the angle $\angle BAC$.

3 Answer parts (a), (b), (c), and *one* of parts (d) and (e). **20%**

. . . .

(a) Use sigma notation to represent the series 2%

$$\frac{1}{3} - \frac{2}{9} + \frac{3}{27} - \frac{4}{81} + \cdots$$

. . . .

(b) The function $\cosh x$ allows the power series representation 5%

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} .$$

Use this series to estimate $\cosh(0.7)$ correct to 4 decimal places.

. . . .

(c) Consider the geometric series: 5%

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$$

Is this series convergent or divergent?

If it is convergent, evaluate its sum.

. . . .

(d) Evaluate the limits: 8%

$$\text{(i)} \quad \lim_{n\to\infty} \frac{2 - n^4 + n}{3n^4 + 1} , \qquad \text{(ii)} \quad \lim_{n\to\infty} \frac{2 + n}{3n^4 + 1} .$$

Hence, apply the Divergence Test to the following series and establish for each of them whether it is divergent or the test is inconclusive:

$$\text{(i)} \quad \sum_{n=1}^{\infty} \frac{2 - n^4 + n}{3n^4 + 1} , \qquad \text{(ii)} \quad \sum_{n=1}^{\infty} \frac{2 + n}{3n^4 + 1} .$$

. . . .

(e) Apply the Ratio Test to the following power series and establish for each value of x whether this series is convergent, divergent, or the test is inconclusive: 8%

$$\sum_{n=1}^{\infty} \frac{(3x)^n}{2^n(n^2 - n + 1)} .$$

4 Answer parts (a), (b), and *one* of parts (c) and (d). **20%**

. . . .

Consider the function

$$f(x) = x^2 - \cos x.$$

(a) Evaluate $f(x)$ from $x = 0.5$ to $x = 1$ using a step of 0.1 and hence sketch a graph of $f(x)$ over the interval $[0.5,\ 1]$.
Does the graph suggest that there is a solution of the equation $f(x) = 0$ between $x = 0.5$ and $x = 1$? (Explain your answer.) 3%

. . . .

(b) Find a root of $f(x) = 0$ to 7 decimal places using the *Newton-Raphson* method with the initial guess $x_0 = 0.7$.
Note that $f'(x) = 2x + \sin x$. 9%

. . . .

(c) Find a root of the equation $f(x) = 0$ to 2 decimal places using the *Bisection* method.
(Hint: use the results of Part (a) to choose the initial interval.) 8%

. . . .

(d) Rewrite the equation $f(x) = 0$ as $x = \sqrt{\cos x}$ over the interval of interest. Hence find a root of this equation to 3 decimal places using the *Fixed Point Iteration* method with the initial guess $x_0 = 0.85$. 8%

USEFUL FORMULAE: $\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$

Angle	0	$\frac{\pi}{6} = 30^o$	$\frac{\pi}{4} = 45^o$	$\frac{\pi}{3} = 60^o$	$\frac{\pi}{2} = 90^o$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0