UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER EXAMINATION PAPER

MODULE CODE: MA4402	SEMESTER: Autumn 2013
MODULE TITLE: Computer Mathematics 2	DURATION OF EXAM: 2 hours 30 minutes
LECTURER: Dr. Niall Ryan	PERCENTAGE OF FINAL GRADE: 70%
EXTERNAL EXAMINER: Prof. T. Myers	

INSTRUCTIONS TO CANDIDATES:

Answer **Question 1** and any other *three* questions from questions **2,3,4,5, and 6**.
Calculators and logarithm tables may be used.
Write all answers in exam scripts. All calculations and work must be shown in order to gain full marks.
Diagrams in the examination paper should be copied into exam scripts.

Q1) i) Use the fast reciprocal algorithm to approximate the value of $\frac{1}{20.13}$ using an initial estimate of 0.05. Halt when the stopping condition is within 0.001 of 1. [3%]

ii) Find the value of a_2 for the following sequence $\begin{cases} a_0 &= 1 \\ a_n &= (n^2-2)a_{n-1} \end{cases}$. [2%]

iii) Evaluate $\lim_{n\to\infty} \frac{6n^2+2n+1}{3n^2+4n+4}$. [2%]

iv) Determine the values of x for which the series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)^2}$ is convergent. [3%]

v) Express 120° in radians, and 5π radians in degrees. [2%]

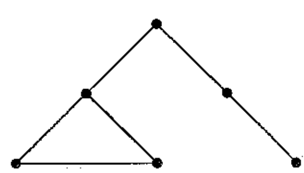
vi) Find the derivative of $3x^5 - 2x^3 + 1$ [2%]

vii) For the vectors $\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, calculate $\mathbf{a} - 2\mathbf{b}$ and $\mathbf{a} \cdot \mathbf{b}$. [2%]

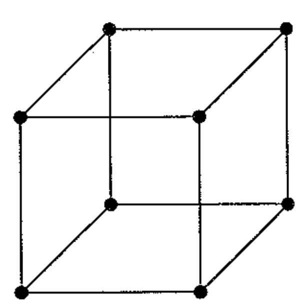
viii) Calculate $-2G^T$ for the matrix $G = \begin{bmatrix} 1 & -1 & -2 & 8 \\ 2 & 4 & 7 & 1 \end{bmatrix}$. [2%]

ix) Find the co-ordinates of the point which results from rotating $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ by $\pi/2$ radians counter-clockwise around the point $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$. [2%]

x) Give the number of vertices and edges in the graph below. In addition, explain why the graph is not a tree. [2%]



xi) Show whether or not the graph below is Eulerian, and whether or not it is Hamiltonian. [3%]



Q2) Bessel's function of order one, $J_1(x)$ can be defined by the infinite power series,

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!} \left(\frac{x}{2}\right)^{2n+1}$$

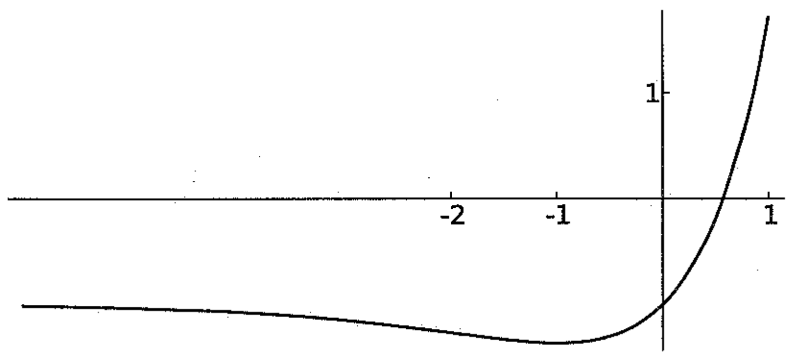
i) Determine the values of x for which this series is convergent. **8%**

ii) Using the series, approximate $J_1(0.2)$ to 4 decimal places. Show your calculations in a table. **7%**

Q3) The function

$$f(x) = xe^x - 1$$

is plotted below.



i) Using Newton's method, approximate the root of $f(x)$ using an initial guess of $x_0 = 1$. Stop when the absolute value of the function at the current estimate becomes less than 0.001. **12%**

ii) Comment on what will occur if $x_0 = 0$, $x_0 = -1$ or $x_0 = -2$ are chosen as initial estimates for the root. **3%**

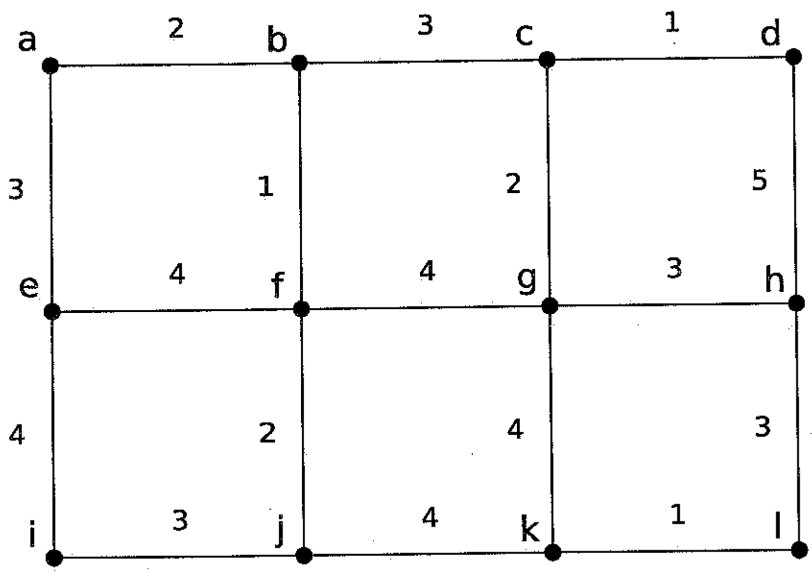
Q4) Consider the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix},$$

i) Rotate the vector $\mathbf{u}$ first by $\pi/2$ radians around the z-axis, then by $2\pi/3$ radians about the y-axis. **8%**

ii) Let $\mathbf{u}_{||}$ be the part of the vector $\mathbf{u}$ which is parallel to the vector $\mathbf{v}$. Calculate the angle between $\mathbf{v}$ and $\mathbf{u} - \mathbf{u}_{||}$ in both radians and degrees. **7%**

Q5) Consider the following weighted graph.



i) Verify Euler's Formula for the complete weighted graph. [5%]

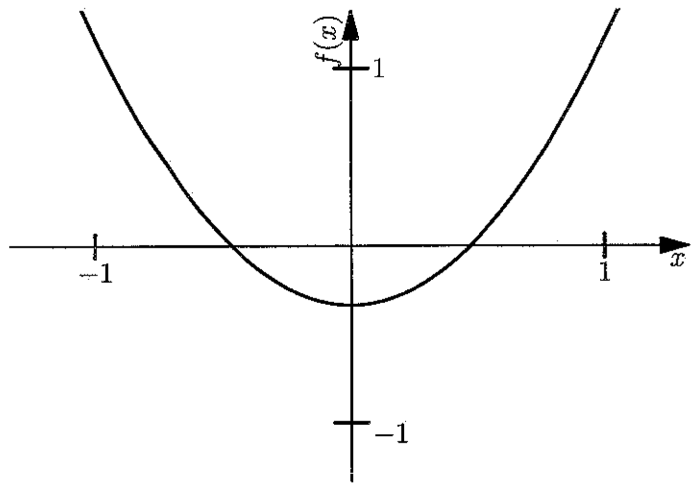
ii) Use Kruscal's algorithm to construct a minimal spanning tree for the graph. List the edges in the order they are added along with their weights. Draw a diagram of the resulting tree. [10%]

Q6) i) The following functions are **not** well defined. [8%]

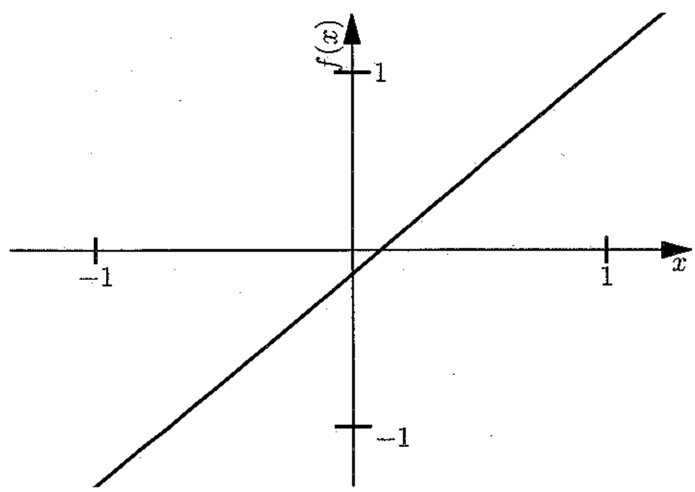
$$f_1 : \mathbb{R} \to \mathbb{R}, \quad f_1(x) = \frac{1}{x - 2013}$$
$$f_2 : \mathbb{R} \to \mathbb{R}, \quad f_2(x) = \sqrt{x + 2013}$$
$$f_3 : \mathbb{R}^+ \to \mathbb{R}^+, \quad f_3(x) = -\sqrt{x}$$
$$f_4 : \mathbb{R}^- \to \mathbb{R}, \quad f_4(x) = \frac{1}{\sqrt{x - 2013}}$$

For each of the functions, without changing its formula, alter the domain and/or co-domain of the function so that it becomes well defined.

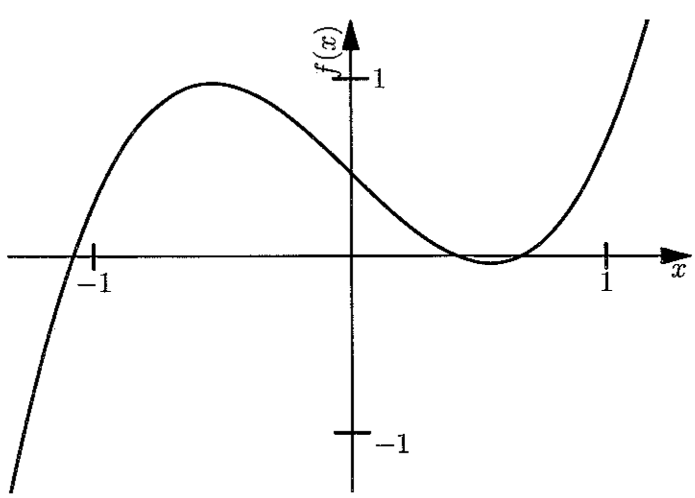
ii) Use the **horizontal line** test to determine whether each of the well defined functions $f : [-1, 1] \to [-1, 1]$ graphed on the next page are i) One to One, (ii) Onto, and/or (iii) Invertible over this domain and co-domain. [7%]



a)



b)



c)

Useful Formulas:

Rules for derivatives:

$f(x)$	$\frac{df}{dx} = f'(x)$	
x^n	nx^{n-1}	For any n
C	0	For any constant C
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$	
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	
e^x	e^x	
e^{ax}	ae^{ax}	For any constant a
$\sin(x)$	$\cos(x)$	
$\cos(x)$	$-\sin(x)$	
$Af(x) + Bg(x)$	$Af'(x) + Bg'(x)$	
uv	$u\frac{dv}{dx} + v\frac{du}{dx}$	

Euler Rotation Matrices:

$$R_z(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix}$$

$$R_y(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix}$$