UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4402	SEMESTER: Autumn 2010
MODULE TITLE: Computer Mathematics 2	DURATION: 2 hours 30 minutes
LECTURER: Niall Ryan	GRADING SCHEME:
EXTERNAL EXAMINER: Dr. T. Myers	Examination 80%

INSTRUCTIONS TO CANDIDATES:

Full marks for correct answers to any 5 questions. All questions carry equal marks. Calculators and logarithim tables may be used.

Q 1 (a) Evaluate the term a_4 in the following sequences. [8%]

(i) $a_n = 2n^2 + 3n - 7$

(ii) $a_n = \begin{cases} n+2 & , \quad n \text{ even} \\ 5^n & , \quad n \text{ odd} \end{cases}$

(iii) $a_n = n! + 3^n$

(iv) $\begin{cases} a_0 = 2 \\ a_n = (a_{n\text{-}1})^n \end{cases}$

(b) Evaluate the limits of the following sequences as $n \to \infty$. [6%]

(i) $a_n = \frac{n}{n+1}$

(ii) $a_n = \frac{10n^2+7n-8}{5n^2-4n-3}$

(iii) $a_n = \frac{(n+1)!}{(n+2)!}$

(c) (i) With an initial guess of 6, use Heron's method to approximate $\sqrt{33}$. Stop iterating when the square of your approximation is within 0.001 of 33. [6%]

(ii) Write Heron's method for approximating $\sqrt{D}$ with initial guess G in the form of a recursive sequence a_n, where $a_0 = G$ is the first term of the sequence. Under the assumption that this recursive sequence has a limit L as $n \to \infty$, show that $L = \sqrt{D}$.

Q 2 (a) For each of the following sequences a_n, evaluate the terms s_2 and s_{21} in the corressponding series $s_n = \sum_{k=0}^{n} a_k$. [8%]

(i) $a_n = 6 + 4n$

(ii) $a_n = 3(2^n)$

(iii) $a_n = \frac{6+4n}{2} + 3(2^n)$

(b) The exponential function is defined using the infinite series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

(i) Use the ratio test to show that this series is convergent for all x. [5%]

(ii) Use the series to estimate the value of $e^{0.15}$ to 3 decimal places, writing your results into the following table. [7%]

n		a_n		s_n
0		.		.
1		.		.
2		.		.
3		.		.
4		.		.

Table for Question 3, part (b)

n		x_n		$f(x_n)$		$f'(x_n)$		$f(x_n)/f'(x_n)$
0		.		.		.		.
1		.		.		.		.
2		.		.		.		.
3		.		.		.		.
4		.		.		.		.
5		.		.		.		.

Spare Table

n		x_n		$f(x_n)$		$f'(x_n)$		$f(x_n)/f'(x_n)$
0		.		.		.		.
1		.		.		.		.
2		.		.		.		.
3		.		.		.		.
4		.		.		.		.
5		.		.		.		.

Q 3 Kepler's problem for an orbit with eccentricity $\frac{1}{2}$ and mean anomaly 1 involves finding the root of the function

$$f(x) = x - \frac{1}{2}\sin(x) - 1$$

where the angle x is measured in **radians**.

(a) Using intervals of 0.5, evaluate f from $x = 0$ to $x = 3$ and hence sketch the graph of $f(x)$ over this interval. [6%]

(b) Given that [12%]

$$f'(x) = 1 - \frac{1}{2}\cos(x)$$

use Newton's method with an initial estimate of 2 to estimate the root of $f(x)$. Stop iterating when at the current estimate x_n, $|f(x_n)| < 0.001$.
Use the table provided on **Page 3**.

(c) Newton's method can be used to estimate $x = \sqrt{D}$, by finding the root of an appropriate function $g(x)$. [2%]

(i) Find an appropriate polynomial function $g(x)$ whose root is $\sqrt{D}$ and

(ii) Find the derivative $g'(x)$ of this function.

Q 4 (a) The following functions are **not** well defined. [8%]

(i) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \frac{1}{x}$

(ii) $f : \mathbb{R} \to \mathbb{R}^+, \quad f(x) = x + 1$

(iii) $f : \mathbb{R} \to \mathbb{R}^+, \quad f(x) = \sqrt[+]{x}$

(iv) $f : \mathbb{R} \to \mathbb{R}^+, \quad f(x) = -(\sqrt[+]{x})$

For each of the functions

(I) Give a counter-example showing that the function is not well defined

(II) Without changing the function's rule, alter the domain and/or codomain of the function so that it becomes well defined.

(b) For each of the 4 graphs shown on **page 5** use the **vertical line** test to determine whether or not each is the graph of a well defined function mapping $\mathbb{R}$ to $\mathbb{R}$. [4%]

(c) The 4 graphs shown on **page 6** are all graphs of well defined functions mapping $\mathbb{R}$ to $\mathbb{R}$. Use the **horizontal line** test to determine whether or not each is function is [8%]

(i) One to One

(ii) Onto

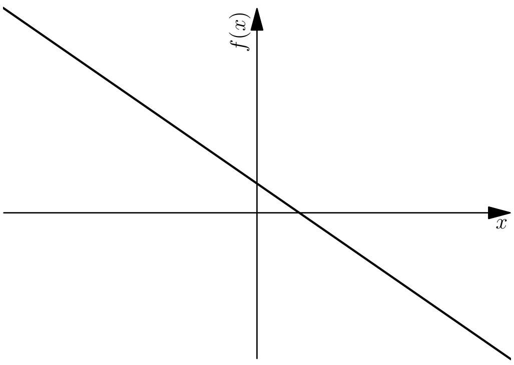
(iii) Invertible

over the domain $\mathbb{R}$ and codomain $\mathbb{R}$.

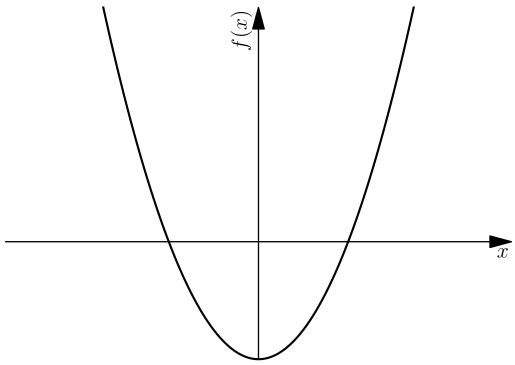
VERTICAL LINE TESTS

Graphs for Question 4, part (b).

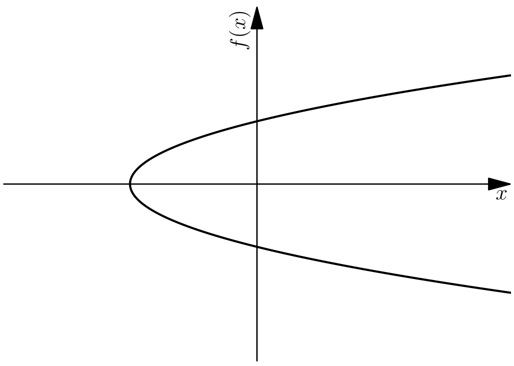
Use the **vertical line** test to determine whether these are the graphs of well defined functions mapping $\mathbb{R} \to \mathbb{R}$.



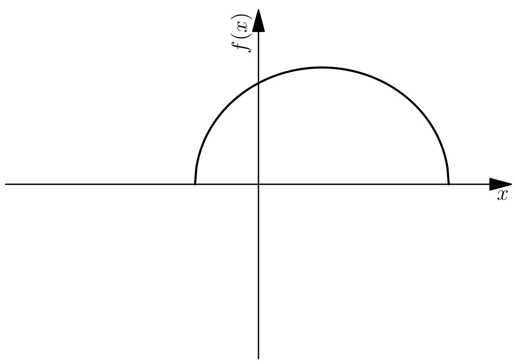
(a)



(b)



(c)



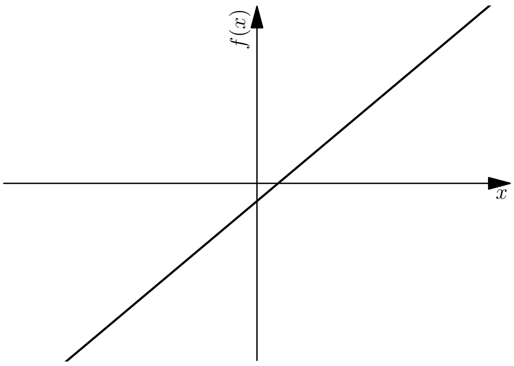
(d)

VERTICAL LINE TESTS

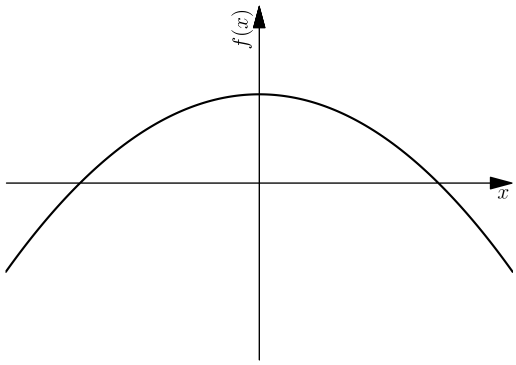
HORIZONTAL LINE TESTS

Graphs for Question 4, part (c).

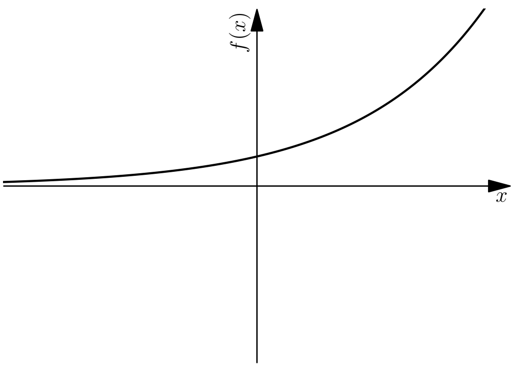
Use the **horizontal line** test to determine whether the well defined functions $f : \mathbb{R} \to \mathbb{R}$ graphed below are (i) One to One, (ii) Onto, and/or (iii) Invertible.



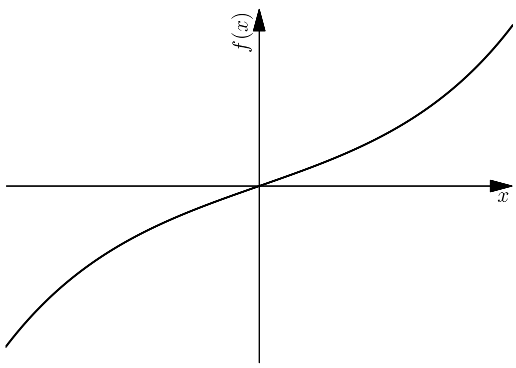
(a)



(b)



(c)



(d)

HORIZONTAL LINE TESTS

Q 5 Consider the following vectors and matrices

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}, A = \begin{bmatrix} 1 & -2 & 1 \\ 7 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

(a) Using the vectors $\mathbf{u}$ and $\mathbf{v}$ [6%]

(I) Evaluate the four expressions

(i) $3\mathbf{u} - 4\mathbf{v}$ (ii) $|\mathbf{u}|$ (iii) $|\mathbf{v}|$ (iv) $\mathbf{u} \cdot \mathbf{v}$

(II) Hence find the angle between $\mathbf{u}$ and $\mathbf{v}$. Express your answer in both radians and degrees.

(b) Using the matrices and vectors above, evaluate the expressions [6%]

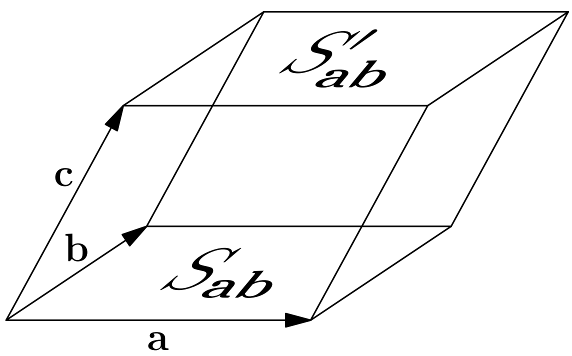
(i) $A\mathbf{u}$ (ii) BC

(c) Rotate the point $\mathbf{x} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ anti-clockwise $\frac{3\pi}{4}$ radians (135°) about the point $\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. [6%]

(d) Calculate the area of the parallelogram which is spanned by the two dimensional vectors $\mathbf{z} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ [2%]

Q 6 Let the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be given by

$$\mathbf{a} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$



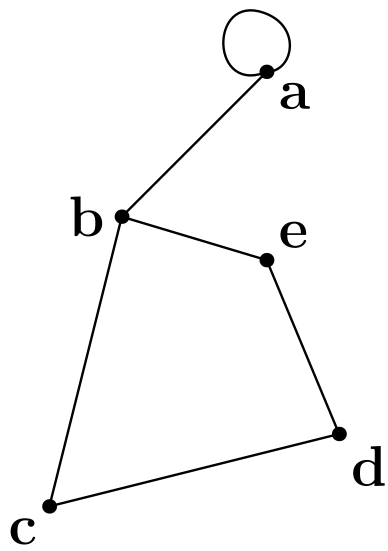
These vectors span a three dimensional parallelepiped as shown in the figure above.

(a) Calculate the volume of the parallelepiped. [5%]

(b) Calculate the area $S_{\mathbf{ab}}$ of the parallelogram spanned by the vectors $\mathbf{a}$ and $\mathbf{b}$. [5%]

(c) Calculate the total surface area of the parallelepiped. [10%]

Q 7 (a) Consider the graph shown to the right. Answer the following questions, and explain your answers by refering the graph shown.



7%

(i) How many nodes and edges are in the graph?

(ii) Is the graph simple?

(iii) Is it connected?

(iv) Give a cycle in the graph.

(v) Write down the adjaceny matrix of the graph.

(b) (i) Draw the graphs K_3 and $K_{3,3}$

5%

(ii) Draw the graph K_4 as a planar graph, and show that it satisfies Euler's formula for planar graphs.

(c) Consider the graphs G_1, G_2, G_3 shown on the next page (**Page 9**). Answer the following questions, and explain your answers by refering to the graphs.

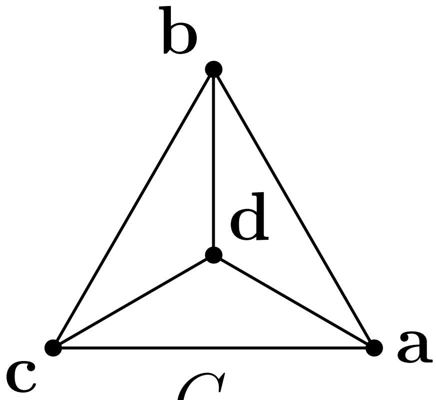
8%

(i) Show that each graph has a Hamiltonian path.

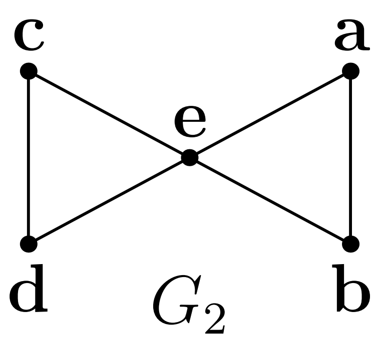
(ii) Show that the graph G_3 is Hamiltonian.

(iii) By finding an Eulerian cycle or otherwise, show whether or not each graph is Eulerian.

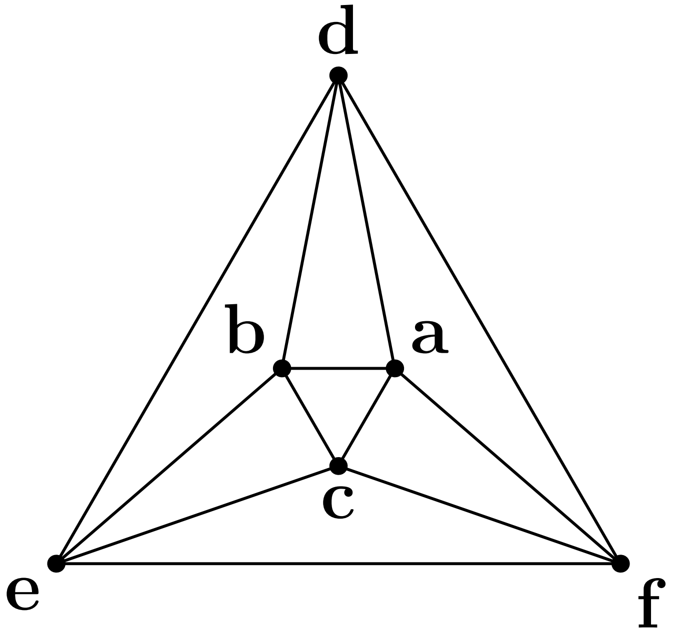
Graphs for Question 7, part (c).



G_1



G_2



G_3