UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4402

SEMESTER: Autumn 2009

MODULE TITLE: Computer Mathematics 2

DURATION OF EXAMINATION: 2hrs 30mins

LECTURER: Dr. S. Soussi

PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES:
Answer 5 questions. All questions carry equal marks.
All questions have to be fully justified.

1. **20%**

(a) A building with N offices is hosting a company having M employees. We name the employees e_1, e_2,..., e_M and $E = \{e_1, e_2, \ldots, e_M\}$ is the set of employees. **10%**

We also name the offices f_1, f_2,..., f_N and $F = \{f_1, f_2, \ldots, f_N\}$ is the set of offices.

Finally we define the function g as follows:

$$g: \ E \ \rightarrow \ F, \ \ g(e_i) = f_j \text{ means that employee } e_i \text{ works in office } f_j.$$

Note that the offices may be unoccupied or occupied by one or more employees.

i. We suppose g injective, compare N and M.

ii. We suppose g surjective, compare N and M.

iii. We suppose g bijective, compare N and M.

(b) Which of the following functions are **10%**

i. injective

ii. surjective

$$\begin{array}{rcccccl} f & : & [0,+\infty) & \rightarrow & \mathbb{R}, & x & \mapsto & x+1, \\ g & : & \mathbb{R} & \rightarrow & [-1,1], & x & \mapsto & \sin(x), \\ h & : & \mathbb{R} & \rightarrow & (0,+\infty), & x & \mapsto & e^x. \end{array}$$

2. **20%**

(a) Give a necessary condition on x so that the series **5%**

$$\sum_{n\geq 1}(-1)^{n+1}\frac{3^n x^n}{n}$$

converges. Note that this series defines $\ln(1+3x)$.

(b) Use the previous series to estimate $\ln(1.3)$ correct to 3 decimal places. **5%**

(c) Find the limit of the sequence $(a_n)_{n\in\mathbb{N}}$ defined by: **5%**

$$a_n = \sin\left(\frac{\pi n + 2}{2n+5}\right).$$

(d) Let $(u_n)_{n\in\mathbb{N}}$ be the recursively defined sequence where: **5%**

$$u_0 = 4, u_1 = 2, \qquad u_{n+2} = \frac{2u_{n+1}+1}{u_n}.$$

We suppose that $(u_n)_{n\in\mathbb{N}}$ is convergent and that $u_n > 0$, for any $n \in \mathbb{N}$. Find the limit l of $(u_n)_{n\in\mathbb{N}}$.

3. Consider the following function: 20%

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 - 3x^2 - 9x + 11.$$

   (a) Find the critical points of f and sketch its graph using this information. 6%

   (b) Deduce from the graph of f the number of roots the function has and, for each root, specify an interval to which it belongs. 6%

   (c) Use the Newton-Raphson method with 5 iterations taking the right choice of the initial guess in order to approximate the largest root. 8%

4. 20%

   (a) What are the characteristics of a simple graph? 4%

   (b) What is the difference between a traversable graph and an Eulerian graph? Give an example of a traversable graph with 4 vertices that is not Eulerian. 4%

   (c) Is $K_{3,3}$ planar? Why? 4%

   (d) Consider the graphs $K_{2,5}$ and $K_{4,4}$. 4%

      i. Draw these graphs naming the vertices v_1, v_2, ...

      ii. Give the adjacency matrix of $K_{2,5}$.

      iii. Which of these 2 graphs are traversable?

      iv. Which of these 2 graphs are Eulerian?

   (e) i. What is the expression tree for 4%

$$(x - y) - \big((x + z) * (y - z)\big).$$

      ii. Is your result a full binary tree? Why?

5. 20%

   (a) Give the rotation matrix of angle θ. 8%

   (b) Consider the line segment with endpoints $(1, 1)$, $(2, 3)$. Give the endpoints of this line segments after rotating it about $(0, 1)$ by $\frac{\pi}{2}$ and then translating it by $(2, -2)$. 12%

6. 20%

   (a) Under which condition on m, n, p, q can we define for the two matrices $M_1 \in \mathbb{R}^{m\times n}$ and $M_2 \in \mathbb{R}^{p\times q}$ 8%

      i. $M_1 + M_2$

      ii. $M_1 M_2$

(b) Let $A = \begin{pmatrix} 2 & -2 & 3 \\ 3 & 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 2 \\ 3 & -1 \\ 4 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix}$. 12%

i. Find when possible AB, BA, AC, CA or say that it is impossible.

ii. Find A^T and B^T.

iii. Show that $B^T A^T = (AB)^T$.