UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4402

SEMESTER: Autumn 2008

MODULE TITLE: Computer Mathematics 2

DURATION OF EXAMINATION: 2hrs 30mins

LECTURER: Dr. S. Soussi

PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES:
Answer 5 questions. All questions carry equal marks.
All questions have to be fully justified.

1. (a) Let $X = \{x_1, x_2, \dots, x_N\}$ be a set of N persons (N is an integer larger than 1) born between the 1st January 1900 and the 31st December 1999. **8%**

   Let U be the set of all possible dates in one year written as a 4 digit number, i.e. the element 1407 of U denotes the 14th of July. Finally let V be the set of all possible dates between the 1st January 1900 and the 31st December 1999 written as a 6 digit number, i.e. the element 140754 of V denotes the 14th of July 1954.

   We define the functions f_1, f_2 as follows:

$$\begin{array}{lcll} f_1 : X & \to & U, & f_1(x) = \text{date of birth of } x \text{ excluding the year}, \\ f_2 : X & \to & V, & f_2(x) = \text{date of birth of } x \text{ including the year}, \end{array}$$

   Note that for simplicity, we suppose all years have 365 days.

   i. Give a necessary condition on N so that f_1 is surjective. Is this condition sufficient?

   ii. Give a necessary condition on N so that f_1 is injective. Is this condition sufficient?

   iii. Give a necessary condition on N so that f_1 is bijective.

   iv. We suppose f_2 injective. What can you say about the persons in X?

   (b) Which of the following functions are **12%**

   A. injective

   B. surjective

$$\begin{array}{ccccccc} f & : & [0, +\infty) & \to & \mathbb{R}, & x & \mapsto & x^3 - 5, \\ g & : & \mathbb{R} & \to & [-1, 1], & x & \mapsto & \cos(x), \\ h & : & \mathbb{R} & \to & (0, +\infty), & x & \mapsto & e^{2x}. \end{array}$$

2. (a) Define the series $(S_n)_{n\in\mathbb{N}}$ corresponding to a given sequence $(u_n)_{n\in\mathbb{N}}$. **3%**

   (b) Give a necessary condition on x so that the series **5%**

$$\sum_{n\geq 1} (-1)^n \frac{2^n x^n}{n}$$

   converges. Note that this series defines $\ln(1 + 2x)$.

   (c) Use the previous series to estimate $\ln(1.2)$ correct to 3 decimal places. **4%**

   (d) Find the limit of the sequence $(a_n)_{n\in\mathbb{N}}$ defined by: **5%**

$$a_n = \cos\left(\frac{\pi n}{2n+1}\right).$$

   (e) Let $(a_n)_{n\in\mathbb{N}}$ be the recursively defined sequence where: **3%**

$$a_0 = -1, a_1 = -2, \qquad a_{n+2} = \frac{1}{a_n} + \frac{1}{a_{n+1}}, \forall n \in \mathbb{N}.$$

   We assume that $(a_n)_{n\in\mathbb{N}}$ is convergent and that $a_n < 0$, $\forall n \in \mathbb{N}$. Find the limit l of $(a_n)_{n\in\mathbb{N}}$.

3. Consider the following function:

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = 4x^3 - 4x^2 - 4x + 1.$$

(a) Find the critical points of f and sketch its graph using this information. [6%]

(b) Deduce from the graph of f the number of roots the function has and for each root, specify an interval to which it belongs. [6%]

(c) Use the Newton-Raphson method with 5 iterations taking the right choice of the initial guess in order to approximate the second root when ordered increasingly. [8%]

4. (a) What are the characteristics of a simple graph? [2%]

(b) What is the difference between a traversable graph and an Eulerian graph? Give an example of a traversable graph that is not Eulerian with 4 vertices. [3%]

(c) What is a Hamiltonian graph? Give an example with 5 vertices and at least 7 edges. [3%]

(d) Consider the graphs $K_{2,4}$, $K_{3,4}$ and $K_{4,4}$. [8%]

i. Draw these graphs naming the vertices v_1, v_2,...

ii. Give the adjacency matrices of $K_{2,4}$.

iii. Which of these 3 graphs are Eulerian?

iv. Which of these 3 graphs are Hamiltonian?

(e) i. What is the expression tree for [4%]

$$(x - y)^2 - (x + z).$$

ii. Is your result a full binary tree? Why?

5. (a) Give the rotation matrix of angle θ. [4%]

(b) Consider the line segment with endpoints $(0, 1)$, $(2, 2)$. Give the endpoints of this line segments after rotating it about $(2, 2)$ by $\frac{\pi}{2}$ and translating it by $(-3, 2)$. [8%]

(c) Consider the following vectors: $u = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $v = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$. Find [8%]

i. $|u|$, $|v|$

ii. $u.v$

iii. $(\widehat{u, v})$

6. (a) Under which condition on m, n, p, q can we define for the two matrices $M_1 \in \mathbb{R}^{m\times n}$ and $M_2 \in \mathbb{R}^{p\times q}$ [8%]

i. $M_1 + M_2$

ii. $M_1.M_2$

(b) Let $A = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 4 \\ 2 & 2 \\ 0 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$. [12%]

i. Find when possible AB, BA, AC, CA or say that it is impossible.
ii. Find A^T and B^T.
iii. Show that $B^T A^T = (AB)^T$.