

Question 1(a)  $\int \frac{x+1}{\sqrt{x+2}} dx \xrightarrow{u} \int \frac{u-1}{\sqrt{u}} du$  } 1%

$$u = x+2$$

$$du = dx$$

$$= \int (\sqrt{u} - u^{-1/2}) du = \frac{2}{3} u^{3/2} - 2 u^{1/2} + C$$
 } 0.5%

$$= \boxed{\frac{2}{3} (x+2)^{3/2} - 2 (x+2)^{1/2} + C}$$
 } 0.5%

Question 1(b)

$$\text{Area} = \int_1^2 (3^x - x^{-3/4}) dx$$
 } 0.5%

$$= \left( \frac{3^x}{\ln 3} - 4 x^{1/4} \right) \Big|_1^2$$

each 0.5%

$$= \boxed{\frac{6}{\ln 3} - 4\sqrt{2} + 4}$$
 } 0.5%

Question 1(c)

$$\{x_i\} = \{1, \dots, 3\}$$

$$\Rightarrow \int_1^3 \frac{1}{(x+1)^3} dx = \frac{(x+1)^{-2}}{-2} \Big|_1^3$$

1%

$$\textcircled{1} = \frac{1}{2} \left( \frac{1}{4} - \frac{1}{16} \right)$$
 } 1%

$$= \boxed{\frac{3}{32}}$$
 } 1%

Question 1(d)

$$\frac{d}{dx} \int_{\sqrt{x+1}}^{2x} \cos(t^4) dx = \frac{d}{dx} \left\{ \int_0^{2x} \cos(t^4) dt \right\} - \frac{d}{dx} \left\{ \int_0^{\sqrt{x+1}} \cos(t^4) dt \right\}$$

$$= \underbrace{2 \cdot \cos(16x^4)}_{0.5\%} - \underbrace{\frac{1}{2\sqrt{x+1}} \cos((x+1)^2)}_{0.5\%}$$

Question 1(e)

$$\frac{d^2}{dx^2}(\cos 3x) = -9 \cos(3x) \Rightarrow M_2 = 9 \quad \left. \vphantom{\frac{d^2}{dx^2}(\cos 3x)} \right\} 0.5\%$$

$$E_T = \frac{1}{12} \frac{(b-a)^3}{h^2} M_2 \quad \longrightarrow 0.5\%$$

$$= \frac{1}{12} \frac{3^3}{h^2} \cdot 9 = \frac{81}{4h^2} \quad \longrightarrow 0.5\%$$

$$\Rightarrow \frac{81}{4h^2} \leq 10^{-2} \Leftrightarrow \frac{9}{2h} \leq \frac{1}{10} \Leftrightarrow \boxed{h \geq 45} \quad \left. \vphantom{\frac{81}{4h^2}} \right\} 0.5\%$$

Question 2

$$\int (2 + \cos x)(\sin^4 x)(\sin x \cdot dx) \quad \textcircled{=}$$

$$\boxed{u = \cos x} \quad \longrightarrow 0.5\%$$

$$du = -\sin x \cdot dx$$

$$\sin^4 x = 1 - u^2$$

$$\textcircled{=} \int (2 + u)(1 - u^2) \overset{\textcircled{2}}{(-du)} = -\int (2 + u - 2u^2 - u^3) du$$

$$= -2u - \frac{u^2}{2} + \frac{2}{3}u^3 + \frac{u^4}{4} + C \quad \left. \vphantom{-2u} \right\} 1\%$$

$$= \boxed{-2 \cos x - \frac{\cos^2 x}{2} + \frac{2}{3} \cos^3 x + \frac{\cos^4 x}{4} + C} \quad \left. \vphantom{-2 \cos x} \right\} 0.5\%$$

Question 3

$$\bar{f} = \frac{1}{7-5} \int_{x=5}^7 \frac{x+3}{x^2-8x+16} dx \quad \left. \vphantom{\int} \right\} 1\%$$

$$x^2 - 8x + 16 = (x-4)^2 \Rightarrow \boxed{u = x-4} \quad \left. \vphantom{\int} \right\} 1\%$$

$$\bar{f} = \frac{1}{2} \int_{u=1}^{u=3} \frac{u+7}{u^2} du \quad \left. \vphantom{\int} \right\} 1\%$$

$$= \frac{1}{2} \left( \ln|u| - \frac{7}{u} \right) \Big|_{u=1}^{u=3} \quad \left. \vphantom{\int} \right\} 0.5\%$$

$$= \boxed{\frac{1}{2} \ln 3 + \frac{7}{3}} \quad \left. \vphantom{\int} \right\} 0.5\%$$

Question 4

$$\int x \underbrace{\ln^2 x}_u dx \quad \textcircled{=}$$

$$dv = x dx \quad \left. \vphantom{\int} \right\} \Rightarrow \quad \begin{aligned} du &= \frac{1}{x} 2 \ln x \\ v &= \frac{x^2}{2} \end{aligned} \quad \left. \vphantom{\int} \right\} 1\%$$

$$\textcircled{=} \frac{x^2}{2} \ln^2 x - \underbrace{\int \frac{x^2}{2} \frac{2 \ln x}{x} dx}_{\parallel} \quad \left. \vphantom{\int} \right\} 1\%$$

$$\int x \underbrace{\ln x}_u dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{dx}{x} \quad \left. \vphantom{\int} \right\} 1.5\%$$

$$\textcircled{=} \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\textcircled{=} \boxed{\frac{x^2}{2} \ln^2 x - \frac{x^2}{2} \ln x + \frac{x^2}{4} + C} \quad 0.5\%$$

Question 5

$$\frac{x^2 - 4x - 8}{x^2(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2} \quad \left. \vphantom{\frac{x^2 - 4x - 8}{x^2(x-2)(x+2)}} \right\} 2\%$$

$$x^2 - 4x - 8 = Ax(x-2)(x+2) + B(x-2)(x+2) + Cx^2(x+2) + Dx^2(x-2)$$

$$\text{Set } x=0 \Rightarrow -8 = B(-4) \Rightarrow \underline{B=2}$$

$$x=2 \Rightarrow -12 = C \cdot 4 \cdot 4 \Rightarrow \underline{C = -\frac{3}{4}}$$

$$x=-2 \Rightarrow 4 = D \cdot 4 \cdot (-4) \Rightarrow \underline{D = -\frac{1}{4}}$$

$$x=1 \Rightarrow -11 = A \cdot (-3) + \underbrace{B(-3)}_{=2} + \underbrace{C(3)}_{=1} + \underbrace{D(-1)}_{=-\frac{1}{4}}$$

each of  
A, B, C, D  
= 0.5%.

$$-11 = -3A - 8$$

$$3A = 3$$

$$\underline{A=1}$$

$$\underbrace{-\frac{9}{4} + \frac{1}{4}}_{= -2}$$

$$I = \int \left( \frac{1}{x} + \frac{2}{x^2} - \frac{3}{4} \frac{1}{x-2} - \frac{1}{4} \frac{1}{x+2} \right) dx$$

$$= \boxed{\ln|x| - \frac{2}{x} - \frac{3}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C}$$

each term = 0.5%