Note: $7 \ln 4 - 3 \ln 2$ instead of $\boxed{11 \ln 2} \to$ ok

3 Find the average value of the function $\frac{4x-6}{x^2-8x+15}$ on the interval $[0,1]$. 4%

(1) complete the square

$x^2 - 8x + 15 = (x-4)^2 - 1$
$u = x - 4$ } 1%

$\bar{f} = \frac{1}{1} \int_0^1 \frac{4x-6}{x^2-8x+15}\,dx$ } $\frac{1}{2}$%

$= \int_{-4}^{-3} \frac{10+4u}{u^2-1}\,du$ } $\frac{1}{2}$%

$= \left(\frac{-10}{2} \ln\left|\frac{u+1}{u-1}\right| + 2\ln|u^2-1|\right)\Big|_{-4}^{-3}$ } $\frac{1}{2}$%

$= 7 \ln|u-1| - 3\ln|u+1| \Big|_{-4}^{-3}$

$= \boxed{11 \ln 2 - 7 \ln 5 + 3 \ln 3} \to \frac{1}{2}$%

(2) partial fractions

$\bar{f} = \frac{1}{1} \int_0^1 \frac{4x-6}{x^2-8x+15}\,dx$ } $\frac{1}{2}$%

$f = \frac{A}{x-5} + \frac{B}{x-3}$ } 1%

$= \frac{7}{x-5} + \frac{-3}{x-3}$ } $\frac{1}{2}$%

$\bar{f} = \left(7\ln|x-5| - 3\ln|x-3|\right)\Big|_0^1$ } $\frac{1}{2}$%

$= \boxed{11 \ln 2 - 7 \ln 5 + 3 \ln 3}$ $\frac{1}{2}$%

4 Evaluate the indefinite integral $\int e^x \underbrace{\cos(2x)}_{u}\,dx$.
(Hint: use integration by parts.) 4%

$I = \cos(2x) \cdot e^x + 2 \underbrace{\int \sin(2x) \cdot e^x\,dx}_{I_*}$, with $u = \sin 2x$ } 1%

$I_* = \sin(2x) \cdot e^x - 2 \underbrace{\int \cos(2x)\, e^x\,dx}_{I}$ } 1%

$\Rightarrow I = \cos(2x) \cdot e^x + 2 \cdot \sin(2x) \cdot e^x - 4I$ } 1%

(3)

$\Rightarrow I = \boxed{\frac{1}{5}\cos(2x) \cdot e^x + \frac{2}{5}\sin(2x) \cdot e^x} + C$ } 1%

Note: similar alternative solution...

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partial marks ok

5 Perform a partial fraction expansion of $\frac{x^2+8x+3}{x^2(x^2+1)}$;
then *evaluate the indefinite integral* $\int \frac{x^2+8x+3}{x^2(x^2+1)}\,dx$. 6%

$f = \underbrace{\frac{A}{x} + \frac{B}{x^2} + \frac{C+Dx}{x^2+1}}_{2\%}$

$\Rightarrow x^2 + 8x + 3 =$
$= Ax(x^2+1) + B(x^2+1) + (C+Dx)x^2$

$\Rightarrow$ $A + D = 0$, $B + C = 1$, $\boxed{A = 8,\ B = 3}$ $\Rightarrow \boxed{D = -8,\ C = -2}$

$\frac{1}{2}$% each

$\Rightarrow f = \frac{8}{x} + \frac{3}{x^2} - \frac{2+8x}{x^2+1}$

$\Rightarrow I = \boxed{8\ln|x| - \frac{3}{x} - 2\tan^{-1}x - 4\ln(x^2+1)} + C$

(4)

$\frac{1}{2}$% each term