1 (a) Evaluate the indefinite integral $\displaystyle\int \frac{3x^{5/4} - 2\sqrt{x}}{x^{3/2}}\,dx.$ 2%

$$= \int 3x^{-1/4} - \int \frac{2}{x}$$

$$= \underbrace{4x^{3/4}}_{1\%} - \underbrace{2\ln x}_{1\%}$$

(b) Calculate the area between $y = \dfrac{1}{\cos^2(x/2)}$ and the x-axis for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. 2%

$$\text{Area} = \int_{-\pi/2}^{\pi/2} \frac{dx}{\cos^2\left(\frac{x}{2}\right)} = 2\int_{-\pi/4}^{\pi/4} \frac{du}{\cos^2 u} \longleftarrow +0.5\%$$

$$= \underbrace{2\tan u \Big|_{-\pi/4}^{\pi/4}}_{+1\%} = 2(1-(-1)) = \underbrace{\boxed{4}}_{+0.5\%}$$

(c) Express as a definite integral and *evaluate* the limit of the Riemann sum $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\cos(3x_i) + \sin(x_i^3)\right) \triangle x,$ where P is the partition with $x_i = -1 + \dfrac{2i}{n}$ for $i = 0, 1, \ldots, n$ and $\triangle x \equiv x_i - x_{i-1}$. 3%

$$= \int_{-1}^{1} \left[\cos(3x) + \sin(x^3)\right] dx \longleftarrow +1\%$$

$$= \frac{1}{3}\sin(3x)\Big|_{-1}^{1} + \int_{-1}^{1} \underbrace{\sin(x^3)}_{\text{odd function}}\,dx$$

$$= \underbrace{\frac{2}{3}\sin 3}_{+1\%} + \underbrace{0}_{+1\%} = \boxed{\frac{2}{3}\sin 3}$$

(d) Evaluate $\dfrac{d}{dx}\displaystyle\int_{2x}^{x^3} \sqrt{\cos(t+1)}\, dt\,.$ 2%

$$= \underbrace{3x^2\sqrt{\cos(x^3+1)}}_{1\%} - \underbrace{2\sqrt{\cos(2x+1)}}_{1\%}$$

(e) Find an upper bound for the error E_T in the Trapezoidal Rule approximation of the definite integral $\displaystyle\int_0^2 \cos(3x)\,dx$, using n subintervals. Choose n such that $E_T \le 10^{-3}$. Hint: evaluate $M_2 \equiv \max\limits_{x\in[0,2]}\left|\dfrac{d^2}{dx^2}\cos(3x)\right|$. 2%

$$M_2 = \max_{[0,2]} 9\,|\cos(3x)| \le 9 \qquad + 0.5\%$$

$$E_T \le \frac{1}{12}\,\frac{(2-0)^3}{n^2}\cdot 9 = \frac{6}{n^2} \qquad + 0.5\%$$

$$\underbrace{\frac{6}{n^2} \le 10^{-3}}_{+0.5\%} \implies \underbrace{\boxed{n \ge 78}}_{+0.5\%}$$

2 Evaluate the indefinite integral $\displaystyle\int \sin^4 x\, \cos^3 x\, dx\,.$ 4%

$$I = \int \sin^4 x\,(1-\sin^2 x)\cos x\, dx$$

$$u = \sin x \qquad 1\%$$

$$= \int u^4(1-u^2)\,du \qquad 1\%$$

$$= \underbrace{\frac{u^5}{5} - \frac{u^7}{7}}_{1\%} = \underbrace{\boxed{\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7}}}_{1\%}$$

3 Find the average value of the function $\ln^2 x$ on the interval $[1, e]$. 5%

$$\bar{f} = \frac{1}{e-1} \int_1^e \underbrace{\ln^2 x}_{u} \cdot \underbrace{dx}_{dv}$$

$$= \frac{1}{e-1} \left(\ln^2 x \cdot x \Big|_1^e - 2\int_1^e \ln x \, dx \right) \text{ — } +2\%$$

$$= \frac{1}{e-1} \left(\ln^2 x \cdot x \Big|_1^e - 2\left\{ \ln x \cdot x \Big|_1^e - \int_1^e dx \right\} \right) +1\%$$

$$= \frac{1}{e-1} \left(\ln^2 x \cdot x \Big|_1^e - 2 \ln x \cdot x \Big|_1^e + 2x \Big|_1^e \right) +1\%$$

$$= \boxed{\frac{e-2}{e-1}} \; +1\%$$

4 Evaluate the definite integral $\displaystyle\int_{-2}^{2} \frac{x+1}{x^2+6x+9}\, dx$. 5%

$$\underbrace{\int_{-2}^{2} \frac{x+1}{(x+3)^2}}_{+1\%} \underset{u=x+3}{=} \underbrace{\int_{u=1}^{u=5} \frac{u-2}{u^2} du}_{+2\%}$$

$$= \int_{u=1}^{u=5} \left(\frac{1}{u} - \frac{2}{u^2} \right) du = \underbrace{\left(\ln u + \frac{2}{u} \right) \Big|_{u=1}^{u=5}}_{+1\%}$$

$$= \boxed{\ln 5 - \frac{8}{5}} \; +1\%$$

<u>Alternative solution</u> : partial fractions for $\dfrac{x+1}{x^2+6x+9}$ ...

5 Perform a partial fraction expansion of $\frac{18x-12}{(x^2-1)(x^2-4)}$.
(but do not integrate this function.) 5%

$$\frac{18x-12}{(x^2-1)(x^2-4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2} + \frac{D}{x+2} \Big\} 2\%$$

$$18x-12 = A(x+1)(x^2-4) + B(x-1)(x^2-4) + C(x+2)(x^2-1) + D(x-2)(x^2-1)$$

$$\left.\begin{aligned}
x=1 &\Rightarrow 6 = A\cdot(-6) \Rightarrow \boxed{A=-1}\\
x=-1 &\Rightarrow -30 = B\cdot 6 \Rightarrow \boxed{B=-5}\\
x=2 &\Rightarrow 24 = C\cdot 12 \Rightarrow \boxed{C=2}\\
x=-2 &\Rightarrow -48 = D\cdot(-12) \Rightarrow \boxed{D=4}
\end{aligned}\right\}$$

any 2 correct ones = 2%, (=1% each),
remaining 2 = 1%. (=0.5% each)

$$\Rightarrow \boxed{\frac{-1}{x-1} - \frac{5}{x+1} + \frac{2}{x-2} + \frac{4}{x+2}}$$