1 (a) Evaluate the indefinite integral $\displaystyle\int \frac{3x - 5x^{1/3}}{\sqrt{x}}\,dx$ 2%

$$\underbrace{-6x^{5/6}}_{1\%} + \underbrace{2x^{3/2}}_{1\%}$$

(b) Calculate the area between $y = \frac{1}{x^2+1}$ and the x-axis for $0 \le x \le 1$. 2%

$$\underbrace{\int_0^1 \frac{dx}{x^2+1}}_{0.5\%} = \underbrace{\tan^{-1} x \Big|_0^1}_{1\%} = \underbrace{\frac{\pi}{4}}_{0.5\%}$$

(c) Express as a definite integral and *evaluate* the limit of the Riemann sum $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(x_i^2 + \sin(\sin x_i)\right)\triangle x$, where P is the partition with $x_i = -1 + \frac{2i}{n}$ for $i = 0, 1, \ldots, n$ and $\triangle x \equiv x_i - x_{i-1}$. 3%

$$\int_{-1}^{1}\left(x^2 + \sin(\sin x)\right)dx =$$

1% (wrong limits $\Rightarrow$ 0.5%)

$$= \underbrace{\frac{2}{3}}_{1\%} + \underbrace{0}_{1\%,}$$

since $\sin(\sin x)$ is odd

(d) Evaluate $\displaystyle \frac{d}{dx}\int_{\sin x}^{1} e^{-(1-t^2)}\,dt\,.$ 2%

$$= -\exp\{-(1-\sin^2 x)\}\cdot(\sin x)' \qquad 1\%$$

$$= -\exp\{\underbrace{-\cos^2 x}_{\text{optional}}\}\cos x \qquad 1\%$$

(e) Find an upper bound for the error E_T in the Trapezoidal Rule approximation of the definite integral $\displaystyle\int_0^1 \sin(2x)\,dx$, using n subintervals, given that $M_2 \equiv \max_{x\in[0,\,1]}\left|\frac{d^2}{dx^2}\sin(2x)\right| = \max_{x\in[0,\,1]}\left|-4\sin(2x)\right| = 4\,.$

Choose n such that $E_T \le \frac{1}{3}10^{-6}$. 3%

$$E_T \le \frac{1}{12}\,\frac{(b-a)^3 M_2}{n^2} = \frac{1}{3n^2} \qquad 1\%$$

$$\frac{1}{3n^2} \le \frac{1}{3}10^{-6} \qquad 1\%$$

$$n \ge 10^3 \qquad 1\%$$

2 Evaluate the indefinite integral $\displaystyle\int \frac{\cos(\ln(t+1))}{t+1}\,dt\,.$ 3%

$$u = \ln(t+1) \qquad 1\%$$

$$= \int \cos u \cdot du \qquad 1\%$$

$$= \sin u = \sin(\ln(t+1)) \qquad 1\%$$

3 Find the average value of $\dfrac{x-2}{x^2+5x+4}$ on the interval $[0,2]$. 5%

$$\bar{f} = \frac{1}{2} \int_0^2 \frac{x-2}{x^2+5x+4}\, dx$$ 1%

(a) partial fr-ns:

$$\bar{f} = \frac{1}{2} \int_0^2 \left(\frac{2}{x+4} - \frac{1}{x+1} \right) dx$$ +2%

$$= \frac{1}{2} \left(2 \ln(x+4) - \ln(x+1) \right) \Big|_0^2$$ +1%

$$= \frac{1}{2} \left(\ln 3 - 2 \ln 2 \right)$$ +1%

(b) complete square: $u = x + \frac{5}{2}$

$$\bar{f} = \frac{1}{2} \int_0^2 \frac{x-2}{(x+\frac{5}{2})^2 - \frac{9}{4}}\, dx = \frac{1}{2} \int_{5/2}^{9/2} \frac{u - 9/2}{u^2 - \frac{9}{4}}\, du = \dots$$

+2% +2%

4 Evaluate the definite integral $\displaystyle\int_0^{\pi/2} x^2 \sin(x)\, dx$. 5%

by parts

$$= \underbrace{-x^2 \cos x \Big|_0^{\pi/2}}_{=0} + \underbrace{\int_0^{\pi/2} 2x \cos x \, dx}$$ +1% +1%

$$\underbrace{2x \sin x \Big|_0^{\frac{\pi}{2}}}_{=\pi} - \underbrace{\int_0^{\pi/2} 2 \sin x \, dx}_{=\underbrace{2 \cos x \Big|_0^{\frac{\pi}{2}}}_{=-2}}$$ +1%

+0.5% +1% +0.5

5 Perform a partial fraction expansion of $\frac{2x-1}{(x+1)(x^2-3x+2)}$. (but do not integrate this function.) 5%

$$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-1}$$ + 2%

$$= \frac{-1/2}{x+1} + \frac{1}{x-2} + \frac{-1/2}{x-1}$$ + 3%

(1% for each A, B, C)