MA4002 ENGINEERING MATHEMATICS 2

Final Exam Comments

In one of the questions you will be requested to give one of the following proofs:

1. Prove that $\int \frac{dx}{x} = \ln|x| + C \quad (x \neq 0)$ from the definition of the indefinite integral (Lecture 1).

2. Prove the Mean-Value Theorem for integrals: If $f(x)$ is continuous on $[a, b]$, then there exists $c \in [a, b]$ such that $f(c) = \bar{f}$, where $\bar{f}$ is the average (mean) value of f on $[a, b]$ (Lecture 4).

3. Prove that $\int_{x_0}^{x_1} f(x)\,dx - y_0 h = -\int_{x_0}^{x_1} (x - x_1) f'(x)\,dx$, where the function $f(x)$ is differentiable on $[x_0, x_1]$, while $h = x_1 - x_2$ and $y_0 = f(x_0)$. (Lecture 13, §13.1).

4. Prove that there exist 2×2 *non-zero* matrices $A,\ B,\ C,$ and D such that

   (i) $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; (ii) $CD \neq DC$;

   (give an example).

5. Prove that

   $$\text{(i)} \qquad \det \begin{bmatrix} a_1 & 0 & 0 & 0 \\ b_1 & b_2 & 0 & 0 \\ c_1 & c_2 & c_3 & 0 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} = a_1\, b_2\, c_3\, d_4 \ ;$$

   $$\text{(ii)} \qquad \det \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ 0 & b_2 & b_3 & b_4 \\ 0 & 0 & c_3 & c_4 \\ 0 & 0 & 0 & d_4 \end{bmatrix} = a_1\, b_2\, c_3\, d_4 \ ;$$

   $$\text{(iii)} \qquad \det \begin{bmatrix} a_1 & 0 & 0 & 0 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & 0 & c_3 & 0 \\ d_1 & 0 & d_3 & d_4 \end{bmatrix} = a_1\, b_2\, c_3\, d_4 \ ;$$

   (Lecture 32).