



UNIVERSITY *of* LIMERICK  
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

**END OF SEMESTER ASSESSMENT PAPER**

MODULE CODE: MA4002

SEMESTER: Spring 2012

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION:  $2\frac{1}{2}$  hours

LECTURER: Dr. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. T. Myers

**INSTRUCTIONS TO CANDIDATES: Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**

**Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.**

**Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.**

**You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.**

- 1 (a) A rocket has acceleration  $a(t) = 20 + 30\sqrt{t}$  metres/second<sup>2</sup> at time  $t$ . It starts from rest at time  $t = 0$ . How far does it travel in the first 4 seconds? 3%
- (b) Consider the plane region bounded by the curve  $y = \frac{1}{x+1}$  and the  $x$ -axis for  $0 \leq x \leq 2$ . Find the volume of each of the two solids obtained by rotating this plane region (i) about the  $x$ -axis; (ii) about the  $y$ -axis. 5%
- (c) Obtain an iterative reduction formula for  $I_n = \int_0^3 (x+1)^n e^{-x/3} dx$ . (Hint: integrate by parts.) Evaluate  $I_0$ . Then, using the reduction formula obtained, evaluate  $I_1$  and  $I_2$ . 4%
- (d) Find all *first and second partial derivatives* of  $f(x, y) = \ln(x + y^2)$ . 4%
- (e) Write down the iterative scheme of the *Improved Euler method* applied to the initial value problem  $y' = \sqrt{x^3 + y}$ ,  $y(0) = 3$  with step size  $h = 0.1$ . Evaluate the approximations of  $y(0.1)$  and  $y(0.2)$  obtained using this scheme. 5%
- (f) Solve the differential equation  $x \frac{dy}{dx} = 2y - 2x^4$ , subject to the initial condition  $y(1) = 5$ . 4%
- (g) Evaluate the determinants  $\begin{vmatrix} 3 & -2 & 7 \\ 4 & 0 & -1 \\ 6 & 1 & -5 \end{vmatrix}$  and  $\begin{vmatrix} 0 & 3 & -2 & 7 \\ -9 & 1 & 8 & -1 \\ 0 & 4 & 0 & -1 \\ 0 & 6 & 1 & -5 \end{vmatrix}$ . 5%
- (Hint: use the first determinant to evaluate the second.)
- (h) To obtain an error estimate for the Rectangular Rule of numerical integration for any differentiable function  $f(x)$ , it is helpful to note that  $\int_{x_0}^{x_1} f(x) dx - y_0 h = -\int_{x_0}^{x_1} (x - x_1) f'(x) dx$ , with the notation  $y_0 = f(x_0)$  and  $h = x_1 - x_0$ . Prove this. (Hint: integrate by parts.) 4%
- .....
- 2 (a) A wineglass is designed by rotating  $y = e^x - 1$ , for  $0 \leq x \leq 1$ , about the  $y$ -axis. Find the volume of the glass. (Hint: use cylindrical shells.) 6%
- (b) A particle moving on the plane in an outward spiral has position vector  $\mathbf{r}(t) = e^t \cos(2t) \mathbf{i} + e^t \sin(2t) \mathbf{j}$ . Find the arc-length along the curve (distance travelled) between times  $t = 0$  and  $t = \pi$ . 6%
- (c) Find the mass and the centre of mass of a rod with mass density  $\rho(x) = \frac{1}{x^2 + 4x + 4}$  for  $0 \leq x \leq 3$ . 6%
- .....

3 (a) Find general solutions of the given differential equations: 3%+3%  
 (i)  $y'' + 6y' + 25y = 0$ , (ii)  $y'' + 3y' = 0$ .

(b) Find a particular solution to the given differential equation: 6%+2%  
 $y'' + 3y' = 10 \sin x - 6$ .  
 Then find the general solution of this equation.

(c) Solve the equation in (b) when  $y(0) = 3, y'(0) = 0$ . 4%

.....

4 (a) Find the Taylor Series, up to and including quadratic terms, 8%  
 of  $z = f(x, y) = (x^2 + y)^{3/2}$  about the point  $(1, 3)$ .

(b) It is known that the quantities  $y$  and  $x$  are related by the formula  $y = kx^\alpha$ , for some unknown constants  $k$  and  $\alpha$ . By writing this as  $\ln y = \alpha \ln x + \ln k$ , one can use the method of least squares to find the best-fit line relating  $\ln y$  to  $\ln x$  and hence find an approximation of the constants  $k$  and  $\alpha$ . For the given data points  
 $(x, y) = (0.5, 1), (1, 2.6), (2, 11), (3, 30), (4, 50)$ ,  
 use this method to find an approximation of the constants  $k$  and  $\alpha$ . 10%

.....

5 (a) Find all solutions of each system of linear equations: 3%+4%  

$$\begin{array}{l} 2x - 6y + 4z = 16 \\ (i) \quad x + 7y - 3z = -22 \\ 5x + 9y - z = -24 \end{array} \quad ; \quad \begin{array}{l} 2x - 6y + 4z = 16 \\ (ii) \quad x + 7y - 3z = -22 \\ 5x + y + 2z = -8 \end{array} .$$

(b) Find the inverse of the matrix 11%  

$$\begin{bmatrix} 2 & -3 & 0 & 0 \\ 4 & -5 & 1 & -1 \\ -2 & 0 & -2 & 0 \\ 0 & 0 & -2 & 7 \end{bmatrix} .$$

.....