UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2009

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) An object has acceleration $a = \sin(0.2t)$ metres/second^2 at time t. The initial velocity at time $t = 0$ is $v = 20$ metres/second. How far does it travel in the first 30 seconds? 4%

(b) Find the volume of the solid obtained by rotating the plane region bounded by the curve $y = \dfrac{1}{x+3}$ and the x-axis for $0 \le x \le 4$, about the x-axis. 4%

(c) Prove that $I_n = \displaystyle\int_0^{\pi/4} \cos^n x\, dx$ satisfies the iterative reduction formula $I_n = \dfrac{1}{n}[2^{-n/2} + (n-1)I_{n-2}]$ for $n \ge 2$. (Hint: integrate by parts with $u = \cos^{n-1} x$ and $dv = \cos x\, dx$.) Evaluate I_0. Then, using the reduction formula obtained, evaluate I_2 and I_4. 4%

(d) Find all *first and second partial derivatives* of $f(x, y) = (xy + 1)e^x$. 4%

(e) Write down the iterative scheme of the *Improved Euler method* applied to the initial value problem $y' = e^{xy}$, $y(0) = 0$ with step size $h = 0.2$. Evaluate the approximations of $y(0.2)$, $y(0.4)$ and $y(0.6)$ obtained using this scheme. 4%

(f) Solve the differential equation $(x^2 + 1)\dfrac{dy}{dx} + x\,y = \dfrac{1}{2}$ with the initial condition $y(0) = 2$. 4%

(g) Evaluate the determinant $\begin{vmatrix} 3 & 5 & -1 \\ -3 & 1 & 7 \\ 1 & 0 & -1 \end{vmatrix}$. 4%

(h) Prove the Mean-Value Theorem for integrals:
If $f(x)$ is continuous on $[a, b]$, then there exists $c \in [a, b]$ such that $f(c) = \bar{f}$, where $\bar{f}$ is the average (mean) value of f on $[a, b]$. 4%

2 (a) Find the volume of the bowl generated by rotating $y = x^2$ for $0 \le x \le 1$, about the y-axis (Hint: use cylindrical shells.) 6%

(b) Find the arc-length along the curve $y = \dfrac{x^4}{32} + \dfrac{1}{x^2}$ for $1 \le x \le 4$. 6%

(c) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \dfrac{1}{(x+1)(4-x)}$ for $0 \le x \le 2$. 7%

3 (a) Find general solutions of the given differential equations: 4%+4%

(i) $y'' + 4y = 0,$ (ii) $y'' + 2y' + y = 0.$

(b) Find a particular solution to the given differential equation: 5%+3%

$y'' + 4y = 4\sin(2x).$

Then find the general solution of this equation.

(c) Solve the equation in (b) when $y(0) = 2,\ y'(0) = 1.$ 3%

4 (a) Find the Taylor Series, up to and including quadratic terms, 9%

of $z = f(x, y) = \dfrac{x-1}{x^2+y}$ about the point $(1, 2)$.

(b) The least squares line approximation to the points 8%+2%

$(-2, 2),\ (-1, 1),\ (0, A),\ (1, -2),\ (2, 4)$ and $(3, B)$ is $y = x + 2$.

(i) Find A and B.

(ii) Sketch the points and the least squares line on the one graph.

5 (a) Find all solutions of each system of linear equations: 4%+3%+3%

(i) $\begin{aligned} x - y + 5z &= 15 \\ 3x - y + 3z &= 11 \\ x + y + 2z &= 17 \end{aligned}$; (ii) $\begin{aligned} x - y + 5z &= 7 \\ 3x - y + 3z &= 7 \\ -x - y + 7z &= 7 \end{aligned}$;

(iii) $\begin{aligned} x - 2y + 3z + 2f &= -3 \\ 3x - y + 4z + f &= 1 \end{aligned}$.

(b) Find the inverse of the matrix 9%

$$\begin{bmatrix} 1 & -4 & -5 \\ 0 & 4 & -2 \\ 2 & -6 & -12 \end{bmatrix}.$$