



UNIVERSITY *of* LIMERICK  
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

**END OF SEMESTER ASSESSMENT PAPER**

MODULE CODE: MA4002

SEMESTER: Spring 2006

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION:  $2\frac{1}{2}$  hours

LECTURER: Dr. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. King

**INSTRUCTIONS TO CANDIDATES: Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**

**Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.**

**Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.**

**You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.**

- 1 (a) A sprinter in a 100-metre race explodes out of the starting block with an acceleration of 4 metre/second<sup>2</sup>, which she sustains for 2 seconds. Her acceleration then drops to zero for the rest of race. What is her time for the race? 4%
- (b) Find the volume of the solid obtained by rotating the plane region bounded by the curve  $y = \frac{1}{x+1}$  and the  $x$ -axis for  $0 \leq x \leq 9$ , about the  $x$ -axis. 4%
- (c) Obtain an iterative reduction formula for  $I_n = \int_0^\pi x^n \cos x \, dx$ . (Hint: integrate by parts twice.) Evaluate  $I_0$ . Then, using the reduction formula obtained, evaluate  $I_2$ ,  $I_4$ , and  $I_6$ . 4%
- (d) Find all *first and second partial derivatives* of  $f(x, y) = x e^{x^2 y}$ . 4%
- (e) Write down the iterative scheme of the *Improved Euler method* applied to the initial value problem  $y' = -xy^2$ ,  $y(0) = 1$  with step size  $h = 0.2$ . Evaluate the approximations of  $y(0.2)$  and  $y(0.4)$  obtained using this scheme. 4%
- (f) Solve the initial value problem  $\frac{dy}{dx} + 2xy = e^{x-x^2}$ , with  $y(0) = 0$ . 4%
- (g) Evaluate the determinant  $\begin{vmatrix} 2 & 7 & 1 \\ 0 & 1 & 5 \\ 4 & 8 & 3 \end{vmatrix}$ . 4%
- (h) Prove the Mean-Value Theorem for integrals:  
If  $f(x)$  is continuous on  $[a, b]$ , then there exists  $c \in [a, b]$  such that  $f(c) = \bar{f}$ , where  $\bar{f}$  is the average (mean) value of  $f$  on  $[a, b]$ . 4%
- 2 (a) A solid of revolution is obtained by rotating about the  $y$ -axis the area bounded between  $y = x \sin x$  and  $y = -\sin x$  for  $0 \leq x \leq \pi$ . Find the volume of the solid obtained. (Hint: use cylindrical shells.) 7%
- (b) Find the arc-length along the curve  $y = \frac{x^3}{24} + \frac{2}{x}$  for  $2 \leq x \leq 4$ . 6%
- (c) Find the mass and the centre of mass of a rod with mass density  $\rho(x) = \frac{x}{x^2 + 4}$  for  $0 \leq x \leq 2$ . 6%

3 (a) Find general solutions of the given differential equations: 4%+4%  
(i)  $y'' - 6y' + 8y = 0$ , (ii)  $y'' - 6y' + 10y = 0$ .

(b) Find a particular solution to the given differential equation: 5%+3%  
 $y'' - 6y' + 8y = 4x e^{2x}$ .  
Then find the general solution of this equation.

(c) Solve the equation in (b) when  $y(0) = 6$ ,  $y'(0) = 3$ . 3%

4 (a) Find the Taylor Series, up to and including quadratic terms, 10%  
of  $z = f(x, y) = (1 + xy) \cos(xy)$  about the point  $(1, 0)$ .

(b) Find the least squares line approximation to the points: 7%+2%  
 $(-4, -4)$ ,  $(-2, -1)$ ,  $(0, 0)$ ,  $(1, 3)$ ,  $(2, 2)$ , and  $(3, 3)$ .  
Sketch the points and the least squares line on the one graph.

5 (a) Find all solutions of each system of linear equations: 4%+4%+2%

$$(i) \begin{aligned} -x + 3y + 4z &= 10 \\ 6x - 8y - 4z &= 20 \\ 3x - 6y &= 24 \end{aligned} ; \quad (ii) \begin{aligned} -x + 3y + 4z &= 10 \\ 6x - 8y - 4z &= 20 \end{aligned} ;$$

$$(iii) -x + 3y + 4z = 10 .$$

(b) Find the inverse of the matrix 9%

$$\begin{bmatrix} -1 & 4 & 3 \\ 2 & -7 & -6 \\ 2 & 0 & -3 \end{bmatrix} .$$