UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2004

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: 2 hours

LECTURER: Dr. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) A car has acceleration $a = \dfrac{d^2s}{dt^2} = e^{-0.2t}$ metres/second^2 at time t. The initial velocity at time $t = 0$ is $v = \dfrac{ds}{dt} = 10$ metres/second. How far does it travel in the first 10 seconds? 4%

(b) Evaluate the given integral or show that it diverges: $\displaystyle\int_0^2 \frac{dx}{x^{2/3}}$. 4%

(c) Find the volume of the solid obtained by rotating the plane region bounded by the curve $y = \sqrt{\sin x}$ and the x-axis for $0 \le x \le \pi$, about the x-axis. 4%

(d) Obtain an iterative reduction formula for $I_n = \displaystyle\int_0^1 x^n e^{-x}\,dx$. (Hint: integrate by parts.) Evaluate I_0. Then, using the reduction formula obtained, evaluate I_2. 4%

(e) Find all *first and second partial derivatives* of $f(x, y) = e^{xy}$. 4%

(f) Write down the iterative scheme of the *Improved Euler method* applied to the initial value problem $y' = -xy$, $y(0) = 1$ with step size $h = 0.4$. Evaluate the approximation of $y(0.4)$ obtained using this scheme. 4%

(g) Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{2\,xy}{1+x^2}$ with $y(0) = -1$. 4%

(h) Evaluate the determinant $\begin{vmatrix} 2 & 1 & 2 \\ 1 & 0 & 3 \\ -1 & 3 & -4 \end{vmatrix}$. 4%

2 Answer part (a) and *any two* of parts (b), (c), (d):

(a) Find the area of the region between $y = \dfrac{2x}{1+x^2}$ and the x-axis, for $0 \le x \le 1$. 5%

(b) A solid of revolution is obtained by rotating about the y-axis the area bounded between $y = \cos x$ and the x-axis for $0 \le x \le \frac{\pi}{2}$. Find the volume of the solid obtained. (Hint: use cylindrical shells.) 7%

(c) Find the arc-length along the curve $y = \dfrac{\ln x}{2} - \dfrac{x^2}{4}$ for $1 \le x \le e$. 7%

(d) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \dfrac{1}{(x+1)(x+2)}$ for $0 \le x \le 2$. 7%

3 (a) Find the general solutions of the given differential equations: 4%+4%

(i) $y'' + 3y' + 2y = 0,$ (ii) $y'' + 4y = 0.$

(b) Find a particular solution to the given differential equation: 5%+3%

$y'' + 3y' + 2y = 4x^2.$

Then find the general solution of this equation.

(c) Solve the equation in (b) when $y(0) = 2,\ y'(0) = 2.$ 3%

4 (a) Find the Taylor Series, up to and including quadratic terms, 10%

of $z = f(x, y) = \dfrac{e^{y+xy}}{x+1}$ about the point $(0, 0)$.

(b) Find the least squares line approximation to the points: 7%+2%

$(-1, 0),\ (0, 1),\ (1, 1),\ (2, 3)$, and $(3, 5)$.

Sketch the points and the least squares line on the one graph.

5 (a) Evaluate the matrix product AA^T, where 2%

$$A = \begin{bmatrix} 1 & 7 & -1 & -4 \\ 2 & -3 & -1 & 0 \end{bmatrix}.$$

(b) Find all solutions of each system of linear equations: 4%+4%

(i) $\begin{array}{rcr} x + 3y + 2z & = & 5 \\ 4x + 9y + 2z & = & -7 \\ -x + y + 4z & = & 19 \end{array}$; (ii) $\begin{array}{rcr} x + 3y + 2z & = & 5 \\ 4x + 9y + 2z & = & -7 \end{array}$

(c) Find the inverse of the matrix 9%

$$\begin{bmatrix} 1 & 4 & 1 \\ -2 & -7 & 1 \\ -3 & -9 & 4 \end{bmatrix}.$$