UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4002

SEMESTER: Spring 2002

MODULE TITLE: Engineering Mathematics 2

DURATION OF EXAMINATION: 2 hours

LECTURER: Dr. E. Gath

PERCENTAGE OF TOTAL MARKS: 70%

EXTERNAL EXAMINER: Prof. J. D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5. To obtain maximum marks you must show all your work clearly and in detail.**
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) A car has acceleration $a = \dfrac{d^2 s}{dt^2} = e^{-0.1t}$ metres/second^2 at time t. It starts from rest at time $t = 0$. How far does it travel in the first 10 seconds? 4%

(b) Sketch the output that results from implementing the Maple command:

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with(student):
rightbox(2*x-1,x=0..3, 3);
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4%

(c) Prove that $I_n \equiv \int \cos^n x\, dx$ satisfies the iterative reduction equation
$I_n = \dfrac{n-1}{n} I_{n-2} + \dfrac{1}{n} \cos^{n-2} \sin x$ for $n \geq 2$.
(Hint: integrate by parts with $u = \cos^{n-1} x$ and $dv = \cos x\, dx$.) 4%

(d) Find the volume when the area enclosed between $y = \cos x$ and the x-axis is revolved about the x-axis for $0 \leq x \leq \frac{\pi}{2}$. 4%

(e) Use partial derivatives and the method of linearisation to find the approximate percentage change in $w = \dfrac{x^5 z^3}{y^2}$ if x increases by 1%, y increases by 3% and z decreases by 2%. 4%

(f) Using the *Euler method*, with step size $h = 0.2$, to approximate $y(0.6)$ where $y(t)$ is the solution of the initial value problem $y' = \tan(xy)$, $y(0) = 0.5$. 4%

(g) Find the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{\cos^2 y}{x}$. 4%

(h) Evaluate the determinant $\begin{vmatrix} 2 & 1 & 1 \\ -1 & -2 & 3 \\ 4 & 3 & 1 \end{vmatrix}$. 4%

2 Answer part (a) and *any two* of parts (b), (c), (d):

(a) Find the area of the region between $y = \sin^3 x \cos^2 x$ and the x-axis, for $0 \leq x \leq \frac{\pi}{2}$. 5%

(b) A solid of revolution is generated by revolving about the y-axis the area bounded between $y = 3x - x^2$ and $y = x$ for $0 \leq x \leq 2$. Find the volume of the solid so obtained. (Hint: use cylindrical shells.) 7%

(c) A particle moving in an outward spiral has position vector $\mathbf{r}(t) = e^t \cos 2t\, \mathbf{i} + e^t \sin 2t\, \mathbf{j}$. Find the arc-length along the curve (distance travelled) between times $t = 0$ and $t = \pi$. 7 %

(d) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \dfrac{4}{x^2 + 1}$, for $0 \leq x \leq 1$. 7%

3 A mass of m kg is attached to a spring and a dashpot. The spring has Hooke constant k N/m. The displacement of the spring at time t seconds is $x(t)$ metres and its velocity $v(t) = \frac{dx}{dt}$ m/s. We assume that the dashpot exerts a damping force $-cv$ N where c is a constant. We may also subject the mass to an external force $F(t)$ N. Applying Newton's second law of motion, the resulting equation of motion is

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t).$$

(a) Consider a mass of $m = 1$ and Hooke constant $k = 9$. If the dashpot is removed and there is no forcing, find the *amplitude* of the oscillation if the initial displacement is $x(0) = 1$ and the initial velocity is $v(0) = 6$. 6%

(b) If a dashpot with constant $c = 6$ is applied to the above system find the displacement at all times $t \geq 0$, subject to the same initial conditions. 7%

(c) Assume the same values of m, k and c and suppose that an external force $F(t) = 27t^2$ is now also applied to the mass. Find the general solution for the displacement $x(t)$ at all times $t \geq 0$. 6%

4 (a) Find the Taylor Series, up to and including quadratic terms, of $z = f(x,y) = \dfrac{xy}{2x+3y}$ about the point $(2,-1)$. 10%

(b) Find the least squares line approximation to the points $(-1,11)$, $(1,8)$, $(2,2)$ and $(5,-4)$. Sketch the points and the least squares line in the one graph. 7+2%

5 (a) Find all solutions of each system of linear equations:

(i)
$$\begin{array}{rrrcl} x & -y & -z & = & 2 \\ x & +3y & +2z & = & 4 \\ & 2y & +3z & = & 4 \end{array}$$

(ii)
$$\begin{array}{rrrcl} x & +2y & -z & = & 1 \\ 2x & & +z & = & 3 \\ 3x & -2y & +3z & = & 5 \end{array}$$

5%+5%

(b) Find the inverse of the 3×3 matrix 9%

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & -2 & 1 \\ -1 & 5 & 0 \end{bmatrix}.$$