University of Limerick

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END–OF–TERM ASSESSMENT

MODULE CODE:	MA4002	DURATION OF EXAM:	$2\frac{1}{2}$hours
MODULE TITLE:	Engineering Maths 2	FRACTION OF TOTAL MARKS:	100%
TERM:	Spring 1999	LECTURER:	Dr. E. Gath

INSTRUCTIONS TO CANDIDATES: Answer question **1** and any three of **2, 3, 4, 5, 6, 7**.

1.(i) [4] An object moves with acceleration $a = 10e^{-\frac{t}{10}}$ at time t. It starts from rest, at time $t = 0$, from position $s = 1000$. Determine its velocity and position at all times $t \geq 0$.

(ii) [4] Evaluate the indefinite integral $\int x^3 \ln x \, dx$.

(iii) [4] Evaluate the limit $\lim_{x \to 4} \left(\frac{1}{x-4} \int_4^x \tan(3t) \, dt \right)$.

(iv) [4] Write down, but do not evaluate, the Riemann sum corresponding to the definite integral $\int_0^\pi \cos x \, dx$, taking the partition P with $x_i = \frac{i\pi}{n}$, for $i = 0, 1, \ldots, n$, and choosing $c_i = \frac{1}{2}(x_{i-1} + x_i)$ (the mid-point of each subinterval).

(v) [4] Write down the MapleVR4 command which gives the *Rectangular Rule* approximation of the integral $\int_0^3 e^{x^2} \, dx$, with 3000 subintervals.

(vi) [4] Find the volume of the solid of revolution that results from revolving about the x-axis, the bounded region enclosed between the curve $y = 5x^2$ and the y-axis, for $0 \leq y \leq 5$.

(vii) [4] Using the *Improved Euler method*, with step size $h = 0.2$, write down an iterative scheme which approximates the solution of the initial value problem $y' = x - y^3, \quad y(0) = 2$.

(viii) [4] The quantities x and y are measured with relative error e_x and e_y respectively. The quantity Q is then calculated from the formula $Q = y\cos(2x + 3y)$. Find the relative error e_Q in terms of e_x, e_y, x and y.

(ix) [4] Solve the initial value problem $\frac{dy}{dx} + 2y = e^{-x}$, with $y(0) = 2$.

(continued over....)

1.(x)[4] For which value of β has the system of equations below an infinite number of solutions?

$$\begin{array}{rrrcl} x & +3y & +2z & = & 1 \\ 2x & -y & +z & = & 1 \\ x & +\beta y & +5z & = & 2 \end{array}$$

2.[6+7+7] Evaluate each of the integrals:

(i) $\displaystyle\int \frac{e^{3t}-e^{-t}}{e^{3t}+3e^{-t}+6}\,dt$ (ii) $\displaystyle\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx$ (iii) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{4-5\sin t}\,dt$

Note re (iii): $\tan\frac{\pi}{8} = \sqrt{2}-1$.

3.[20] Attempt any *three* of parts (i), (ii), (iii), (iv).

(i) Find the area enclosed between $y = 2\cosh x$ and $y = e^x$ for $x \geq 0$.

(ii) Find the volume of the solid of revolution that results from revolving about the y-axis, the region enclosed between the curve $y = 14 - 3x - x^3$ and the x-axis, for $0 \leq x \leq 2$.

(iii) Find the arc-length along the curve $x = \dfrac{y^3}{6} + \dfrac{1}{2y}$ for $1 \leq y \leq 2$.

(iv) Find the mass and the centre of mass of a rod with mass density $\rho(x) = \sqrt{4-x^2}$, for $0 \leq x \leq 1$.

4.(a)[10] Prove that $I_n \equiv \displaystyle\int_0^{\frac{\pi}{2}} (\cos x)^{2n+1}\,dx$, $n = 0, 1, 2, 3, \ldots$, satisfies the iteration

$I_{n+1} = \left(\dfrac{2n+2}{2n+3}\right) I_n$. Hence prove that $I_n = \dfrac{2^{2n}\,(n!)^2}{(2n+1)!}$.

(b)[10] Find the least squares line approximation to the points $(1,4)$, $(2,4)$, $(3,7)$, $(5,8)$, $(7,8)$. Plot these points and the line in the same graph.

5.(a)[8] Find the Taylor Series, up to and including quadratic terms, of $z = f(x, y) = y^2 e^x + \sin(3x + y)$ about the point $(0, \pi)$.

(b)[6+4+2] Prove that $M_2 \equiv \max_{x\in[0,1]} \left| \frac{d^2}{dx^2} \ln(x^2 + 1) \right| = 2$, that is the maximum value of the second derivative of $\ln(x^2 + 1)$, on the interval $0 \leq x \leq 1$, is 2.

Find an upper bound on the error for the *Trapezoidal Rule* approximation of the definite integral $\int_0^1 \ln(x^2 + 1)\, dx$ using n subintervals.
How many subintervals would be required to ensure an error of less than 10^{-10}?

6. The charge $q(t)$, at time t, in the capacitor in the LRC circuit depicted below satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t).$$

Consider a circuit with an inductor with $L = 1$ Henry, a resistor with $R = 2$ ohms and a capacitor with $C = 0.2$ Farads.

(a)[8] Write down the general solution to the homogeneous equation *i.e.* when the external voltage $E(t) = 0$. Sketch a typical solution.

(b)[8] By first finding a particular solution, find the general solution to the differential equation when the external voltage $E(t) = 5 \sin t$.

(c)[4] Solve the equation in **(b)** when the initial charge on the capacitor is $q(0) = 1$ Farad and the initial current is $q'(0) = 0$ amps.

7. (a)[6] Write down a system of four linear equations in two unknowns

(i) which is inconsistent;

(ii) which has a unique solution;

(iii) which has an infinite number of solutions.

(b)[8+3+3] Find the inverse matrix A^{-1} of the 3×3 matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & -1 \end{bmatrix}.$$

Find the matrix AA^{-1}. Evaluate the determinant $\det A$.