University of Limerick

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College of Informatics and Electronics

END–OF–TERM ASSESSMENT

MODULE CODE:	MA4002	DURATION OF EXAM:	$2\frac{1}{2}$ hours
MODULE TITLE:	Engineering Maths 2	FRACTION OF TOTAL MARKS:	100%
TERM:	Spring 1998	LECTURER:	Dr. E. Gath

INSTRUCTIONS TO CANDIDATES: Answer question **1** and any three of **2, 3, 4, 5, 6, 7**.

1.(i) [4] A tap drips continuously at the rate of $\frac{200}{(t+3)^3}$ cubic centimeters/second. Determine the total volume that drips from the tap between times $t = 0$ and $t = 7$.

(ii) [4] Evaluate the indefinite integral $\int \frac{x}{x^2 - 3x + 2}\, dx$.

(iii) [4] Find $\frac{dy}{dx}$ when $y = \int_0^{3x} \cos(t^2)\, dt$.

(iv) [4] Find the average value of $y = te^{-t}$ on the interval $0 \le t \le 2$.

(v) [4] Sketch the output which results from implementing the MapleV4 command:

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with(student):
rightbox(2-x, x=0..2, 4);
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(vi) [4] Express as a definite integral, and hence evaluate, the limit of the Riemann sum $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{c_i}\, \Delta x_i$, where P is the partition with $x_i = 1 + \frac{3i}{n}$, for $i = 0, 1, \ldots, n$, $\Delta x_i \equiv x_i - x_{i-1}$ and $c_i \in [x_{i-1}, x_i]$.

(vii) [4] Find all *first and second partial derivatives* of $f(x, y) = \tan(xy)$.

(viii) [4] Solve the initial value problem $\frac{dy}{dx} = \frac{y^2}{x}$, with $y(1) = 1$.

(ix) [4] Use the *Euler method* to write down an iterative approximation of the solution of the initial value problem $y' = (x + y)^2$, $y(0) = 1$, choosing step size $h = 0.2$.

(x) [4] Evaluate the determinant $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{vmatrix}$.

2.[6+7+7] Evaluate each of the integrals:

(i) $\int_0^{\frac{\pi}{4}} \frac{\sin t \cos t}{1 + 2\cos^2 t}\, dt$ (ii) $\int_0^1 \frac{x^2}{x^2 - 6x + 10}\, dx$ (iii) $\int \sqrt{x}\,(\ln x)^2\, dx$

3.[20] Attempt any *three* of parts (i), (ii), (iii), (iv).

(i) Find the area enclosed between $y = \sec x$ and $y = \sqrt{2}$ (-specifically the area of the component containing the point $(0, 1)$).

(ii) Find the volume of the solid of revolution that results from revolving the region, enclosed by the curve $x = y^2 - 3y$ and the y-axis, about the y-axis.

(iii) Find the mass of a rod with mass density $\rho(x) = \rho_0(2 + \sqrt{x})$, for $0 \le x \le 4$.
Find the moment of inertia of this rod when it is rotating about the point $x = 1$.

(iv) A particle has position vector $\mathbf{r}(t) = e^{-t} \sin 2t\, \mathbf{i} + e^{-t} \cos 2t\, \mathbf{j}$ at time t. Find the distance travelled by the particle between times $t = 0$ and $t = 1$.

4. (a)[10] Given that $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$, find $\int_0^\infty x^2 e^{-x^2}\, dx$. (Hint: use integration by parts with $u = x$ and $dv = xe^{-x^2}\, dx$.)
Prove that $I_n \equiv \int_0^\infty x^{2n} e^{-x^2}\, dx$ satisfies $I_n = \left(\frac{2n-1}{2}\right) I_{n-1}$ for all $n = 1, 2, 3, \ldots$.

Evaluate I_2 and I_3. Prove that $I_n = \frac{(2n)!\sqrt{\pi}}{2^{2n+1}\, n!}$ for all $n = 1, 2, 3, \ldots$.

(b)[10] Find the Taylor Series, up to and including quadratic terms,
of $z = f(x, y) = \ln(x^2 + y^2) + \cosh(x - y)$ about the point $(1, 1)$.

5.(a)[10] The least squares line approximation to the points
$(0, 4)$, $(1, 0)$, $(2, a)$, $(5, b)$, $(7, -27)$ is $y = \frac{23}{5} - 4x$. Find a and b.

(b)[10] Use *Simpson's rule*, with four equal subintervals (*i.e.* $n = 2$), to find an approximation for the definite integral $\int_0^1 \cos(e^x)\, dx$. Assuming that $M_4 \equiv \max_{x \in [0,1]} \left| \frac{d^4}{dx^4} \cos(e^x) \right| < 60$,
find an upper bound for the error in the above approximation.
How many subintervals would be required to ensure an absolute error of less than 10^{-8}?

6.(a)[10] The current $i(t)$, at time t, in an LR circuit with constant external voltage E_0 satisfies the first order linear differential equation

$$L\frac{di}{dt} + Ri = E_0.$$

Suppose the initial current $i(0)$ is zero. By finding an appropriate integrating factor, solve this differential equation to find $i(t)$ for all $t \geq 0$ and show that as $t \to \infty$, $i(t)$ tends to its constant Ohm's law value.

(b)[10] Find the general solution of the nonhomogeneous linear second order constant coefficient ordinary differential equation $y'' + 2y' + y = 2e^{-x}$.

7.(a)[5+5] Use the method of *Gauss-Jordan elimination* to find all solutions of each system of linear equations:

(i)
$$\begin{array}{rrrcl} x & +y & -2z & = & 1 \\ 3x & -y & +z & = & 4 \\ x & -3y & +5z & = & 3 \end{array}$$

(ii)
$$\begin{array}{rrrcl} x & +3y & -4z & = & 0 \\ x & +2y & +z & = & 1 \\ 3x & +7y & -3z & = & 2 \end{array}$$

(b)[10] Find the inverse of the 3×3 matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 4 & 1 \\ -2 & 9 & 1 \end{pmatrix}.$$