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ON THE STABILITY OF TWO-LAYERED LARGE-AMPLITUDE GEOSTROPHIC FLOWS WITH THIN UPPER LAYER

EUGENE S. BENILOV

School of Mathematics, University of New South Wales, P.O. Box 1, Kensington, NSW 2033, Australia

BENOIT CUSHMAN-ROISIN

Thayer School of Engineering, Dartmouth College, Hanover, NH 03755, USA

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We examine the stability of two-layer geostrophic flows with large displacement of the interface. The depth of the upper layer is assumed much smaller than the total depth of the fluid. It is proven that all westward and weak eastward flows are stable with respect to disturbances whose wavelengths is of the order of, or longer than the width of the flow. It is further demonstrated that westward flows are (locally) unstable with respect to short disturbances.

KEY WORDS: β-plane, geostrophic flows, baroclinic instability.

1. INTRODUCTION

Stability of oceanic fronts and density-driven currents is an important problem of physical oceanography, however, the full equations of geophysical fluid dynamics are very complex and, generally speaking, do not allow analytical study. On the other hand, the traditional quasi-geostrophic approach is not applicable to fronts either, as the vertical displacement of isopycnal surfaces in frontal flows is large. Nevertheless, the Rossby number for most of real-ocean fronts and currents is small, which enabled Williams and Yamagata (1984), Cushman-Roisin *et al.* (1992) and Benilov (1992a) to modify the geostrophic formalism for *large*-amplitude geostrophic flows in a two-layer fluid and derive relatively simple asymptotic governing equations. Using these equations, Benilov (1992a, b) and Swaters (1993) examined the stability of zonal flows with horizontal shear. It should be emphasized, however, that the dynamics of large-amplitude geostrophic flows depend strongly on the correlation between the following non-dimensional parameters:

1) the Rossby number

$$\varepsilon = U/fL,$$

where U is the effective velocity scale, f is the Coriolis parameter and L is the horizontal spatial scale of the motion;

2) the ratio of the depth of the upper layer to the total depth of the ocean

$$\delta = H_1/H;$$

3) the β-effect number

$$\beta = (R_d/R_e)\cot\theta,$$

where R_e is the earth's radius, θ is the latitude,

$$R_d = \sqrt{g'H}/f$$

and $g' = g(\Delta\rho/\rho)$ is the reduced acceleration due to gravity (compare R_d to the standard deformation radius given by $R'_d = \sqrt{g'H_1}/f$). Accordingly, the above results on the stability of zonal flows can be classified in the form of a table:

	Weak β-effect: $\beta \sim \varepsilon^{3/2}$	*Strong β-effect*: $\beta \sim \varepsilon$
$\delta \sim 1$	Benilov (1992a)	Benilov (1992a)
$\delta \sim \varepsilon$		
$\delta \sim \varepsilon^2$	Swaters (1993)	Benilov (1992b)

It is worth noting that the case $\delta \sim \varepsilon^2$ is hardly relevant to any real-ocean situation: given that the flow is geostrophic and $\varepsilon \leqslant 0.1$, this condition entails the (absolutely unrealistic) constraint $\delta \leqslant 0.01$ (the typical ocean values are $H \sim 2{,}000$–$6{,}000$ m, $H_1 \sim 200$–800 m, $\delta \sim 1/3 - 1/15$). At the same time, there are two gaps in the table at $\delta \sim \varepsilon$, and these regimes are quite possible in the ocean [for a detailed discussion of the parameter space of the problem see Cushman-Roisin *et al.* (1992)].

In this paper, we shall consider the stability of large-amplitude geostrophic flows with both horizontal and vertical shear for the case $\delta \sim \varepsilon$.

2. FORMULATION OF THE PROBLEM

Consider a geostrophic

$$\varepsilon \ll 1 \tag{1}$$

flow of a two-layered fluid on the β-plane. If the interface displacement is of the order of the depth of the upper layer:

$$\delta H_1 \sim H_1 \tag{2}$$

and the upper layer is much thinner than the total depth of the ocean:

$$\delta \ll 1, \tag{3}$$

the governing shallow-water equations can be reduced (Cushman-Roisin *et al.*, 1992) to

$$\left.\begin{aligned} &h_t + J(p,h) = \nabla\cdot[h\,J(h,\nabla h)] + \beta h h_x, \\ &\Delta p_t + J(p,\Delta p) + \beta(p + \tfrac{1}{2}h^2)_x + \nabla\cdot[h\,J(h,\nabla h)] = 0; \end{aligned}\right\} \tag{4}$$

where we have defined

$$h = \tilde{h}/H, \qquad p = \tilde{p}/g'H,$$

$$t = \tilde{t}f, \qquad x = \tilde{x}/R_d, \qquad y = \tilde{y}/R_d$$

and where the dimensional variables (the time $\tilde{t}$, the horizontal spatial variables $(\tilde{x}, \tilde{y})$, the depth of the upper layer $\tilde{h}$ and the pressure in the bottom layer $\tilde{p}$) are marked with tildas.

It should be emphasized that conditions (1)–(3) restrict the horizontal spatial scale of the flow. Indeed, if we substitute the geostrophic velocity scale

$$U = \frac{1}{f} g' \frac{\delta H_1}{L}$$

into (1) and take into account (2), we obtain

$$\frac{g' H_1}{f^2 L^2} \ll 1,$$

which can be rewritten as

$$\left(\frac{L}{R_d}\right)^2 \gg \frac{H_1}{H}. \tag{5}$$

This inequality is the main restriction of our results.

In order to further simplify system (4), we shall separate the cases of strong and weak β-effect and eliminate the (unrealistic) regimes with $\delta \lesssim \varepsilon^2$. A straightforward asymptotic analysis yields

weak β-effect: $\beta \sim \varepsilon^{3/2}$, $\delta \sim \varepsilon$ $\quad (h \sim \varepsilon, p \sim \varepsilon^{-3/2}, t \sim \varepsilon^{-3/2}, x \sim y \sim 1)$,

$$\left.\begin{aligned} &h_t + J(p,h) = 0, \\ &\Delta p_t + J(p,\Delta p) + \beta p_x + \nabla\cdot[h\,J(h,\nabla h)] = 0; \end{aligned}\right\} \tag{6}$$

strong β-effect: $\beta \sim \varepsilon$, $\delta \sim \varepsilon$ $\quad (h \sim \varepsilon, p \sim \varepsilon^2, t \sim \varepsilon^{-2}, x \sim y \sim 1)$,

$$\left.\begin{aligned} & h_t + J(p,h) = \nabla\cdot[h\, J(h, \nabla h)] + \beta h h_x, \\ & \beta(p + \tfrac{1}{2}h^2)_x + \nabla\cdot[h\, J(h,\nabla h)] = 0. \end{aligned}\right\} \tag{7}$$

Both systems (6) and (7) admit the following steady solution:

$$h(x,y,t) = H(y), \quad p(x,y,t) = P(y); \;\; H, P \to \text{const} \quad \text{as} \quad y \to \pm\infty; \tag{8}$$

which describes a localized zonal flow with both horizontal and vertical shear.

In the next section we shall discuss the stability of solution (8) within the framework of systems (6) and (7).

3. STABILITY OF FLOWS WITH THIN UPPER LAYER

3.1 *The case of weak β-effect*

It should be noted that system (6) coincides with the $(h \to 0)$ limit of the corresponding system for flows with "thick" $(h \sim 1)$ upper layer [see equations (12) in Benilov, 1992a]. Emulating the results obtained by Benilov (1992a), we conclude that *in the case of weak β-effect all zonal flows with thin upper layer are unstable.*

In order to illustrate this conclusion, we shall derive the stability boundary-value problem for equations (6) and solve it for a particular case of frontal flow.

Following the standard procedure, we linearize equations (6) against the background of steady solution (8), i.e. substitute

$$h(x,y,t) = H(y) + h'(x,y,t), \qquad p(x,y,t) = P(y) + p'(x,y,t)$$

into (6) and neglect the nonlinear terms. Substituting then

$$h'(x,y,t) = h(y)\exp[ik(x-ct)], \qquad p'(x,y,t) = p(y)\exp[ik(x-ct)],$$

where c and k are the phase speed and wavenumber of a disturbance, we obtain

$$\left.\begin{aligned} & (c + P_y)h - H_y p = 0, \\ & (c + P_y)(p_{yy} - k^2 p) - P_{yyy}p - \beta p \\ & \qquad + [H(H_y h_y - H_{yy} h)]_y - k^2 H H_y h - \beta H h = 0. \end{aligned}\right\} \tag{9}$$

Eliminating p and introducing a new variable ϕ such that

$$h = H_y \phi, \tag{10}$$

we write system (9) in the form

$$(F\phi_y)_y - k^2 F\phi - \beta(c + P_y)\phi = 0, \tag{11a}$$

where

$$F = (c + P_y)^2 + H(H_y)^2. \tag{11b}$$

We shall look for perturbations localized in the vicinity of the flow:

$$\phi \to 0 \quad \text{as} \quad y \to \pm\infty. \tag{12}$$

The boundary-value problem (11), (12) determines the eigenvalues c. The thick-upper-layer analogue of equation (11) was derived by Benilov (1992a) and differs from (11) only in the expression for F:

$$F = (c + P_y)^2 + H(1 - H)(H_y)^2$$

[which coincides with (11b) in the limit $H \ll 1$]. Accordingly, the proof of instability by Benilov (1992a) can be easily generalized (or, rather, emulated) for our case.

In order to illustrate the instability, consider the following frontal flow (Figure 1):

$$H(y) = \begin{cases} \gamma^{2/3} & \text{for} \quad y \in (-\infty, 0], \\ (\gamma + \alpha y)^{2/3} & \text{for} \quad y \in [0, l], \\ (\gamma + \alpha l)^{2/3} & \text{for} \quad y \in [l, \infty); \end{cases} \qquad P(y) = \begin{cases} 0 & \text{for} \quad y \in (-\infty, 0], \\ -uy & \text{for} \quad y \in [0, l], \\ -ul & \text{for} \quad y \in [l, \infty); \end{cases} \tag{13}$$

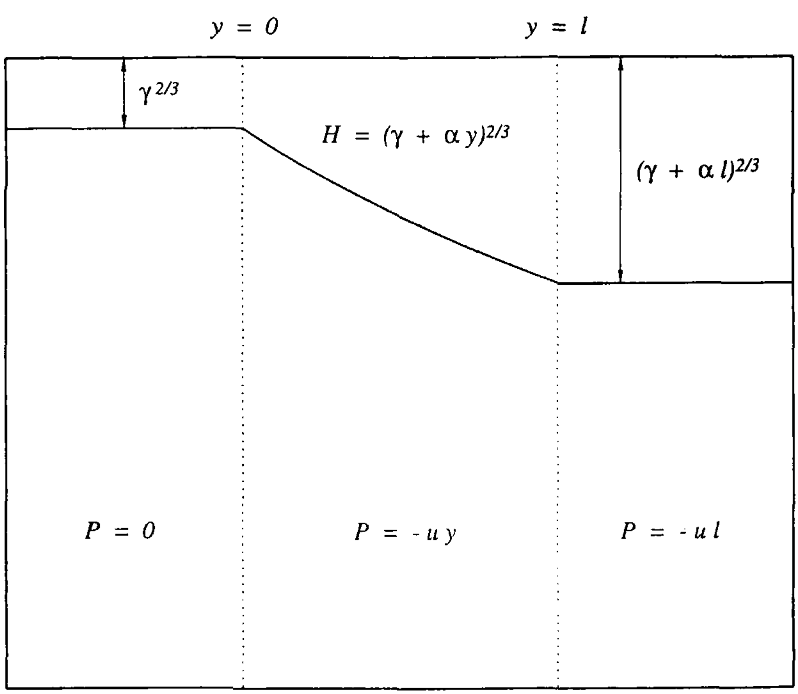

Figure 1 Frontal flow (13).

where l is the (non-dimensional) width of the flow, u is the velocity in the bottom layer, and the constants α and γ characterize $H(y)$ for $y \leqslant 0$ and $y \geqslant l(\alpha \geqslant -\gamma/l)$. Substitution of (13) into (11) yields and equation with constant coefficients (which is why we chose this particular case):

$$(F\phi_y)_y - k^2 F\phi - \beta(c-u)\phi = 0, \tag{14a}$$

$$F = (c-u)^2 + \tfrac{4}{9}\alpha^2. \tag{14b}$$

As the perturbation does not penetrate beyond the boundaries of the flow, the boundary conditions should be rewritten as follows:

$$\phi \to 0 \quad \text{at} \quad y = 0, l. \tag{15}$$

The boundary-value problem (14), (15) can be readily solved:

$$\phi = \sin\left(\frac{\pi m}{l} y\right), \quad m = 1, 2, 3, \ldots$$

$$c = u + \frac{-\beta \pm \sqrt{\beta^2 - \frac{16}{9}\alpha^2\left[k^2 + \left(\frac{\pi m}{l}\right)^2\right]^2}}{2\left[k^2 + \left(\frac{\pi m}{l}\right)^2\right]}. \tag{16}$$

Formula (16) yields instability ($\operatorname{Im} c \neq 0$) for

$$\sqrt{\frac{3\beta}{4|\alpha|} - \left(\frac{\pi}{l}\right)^2} < k < \infty \tag{17}$$

and demonstrates that the growth rate grows with k and m:

$$k \operatorname{Im} c \to \tfrac{2}{3}\alpha k \quad \text{as} \quad k, m \to \infty.$$

This unbounded growth is a result of inapplicability of (16) for $k^2 \gtrsim \varepsilon^{-1}$ [see restriction (6) with $H_1/H = \varepsilon$ and $L \sim R_d/k$]. Thus, formula (16) [and the original system (4)] fails to describe the most unstable perturbations, whose wavelengths are comparable to the upper-layer deformation radius

$$L \sim R_d' = \sqrt{g' H_1}/f = R_d \varepsilon^{1/2}$$

and correspond to $k^2 \sim \varepsilon^{-1}$ (e.g. Killworth *et al.*, 1984; Benilov, 1992a). However, (4) and (16) did enable us to establish the fact of instability and calculate the long-wave limit of the growth rate. Accordingly, estimate (17) for the spectral margins of the instability should be modified:

$$k_{\text{mar}} < k \lesssim \varepsilon^{-1/2},$$

where

$$k_{\text{mar}} = \sqrt{\frac{3\beta}{4|\alpha|} - \left(\frac{\pi}{l}\right)^2}.$$

Evidently,

$$k_{\text{mar}} \to \infty \qquad \text{as} \qquad \beta \to \infty,$$

which indicates that, in the transitional interval between the regimes of weak and strong β-effect:

$$\varepsilon^{3/2} \lesssim \beta \lesssim \varepsilon,$$

all perturbations with wavelengths comparable to the width of the flow are stable and the instability takes place at short wavelengths.

3.2 *The case of strong β-effect*

As system (7) does not coincide with the $(h \to D)$ limit of the corresponding thick-upper-layer equations [compare (7) to equations (14) in Benilov (1992a)], we cannot call on the corresponding results here. However, the structure of the two systems is similar, and we can employ the same approach.

Following the standard procedure, we shall linearize equations (7) against the background of steady solution (8), assume the harmonic dependence of h and p on x and t and introduce, according to (10), ϕ instead of h:

$$\frac{\beta}{\beta + H_y}(c + P_y)H_y\phi = [H(H_y)^2\phi_y]_y - k^2 H(H_y)^2\phi - \beta H H_y \phi. \tag{18}$$

[the thick-upper-layer analogue of this equation is

$$(c + P_y)\phi = [H(1-H)(H_y)^2\phi_y]_y - k^2 H(1-H)(H_y)^2\phi - \beta H(1-H)H_y\phi$$

—see Benilov (1992a)]. Now, multiplying (18) by ϕ^* (where * denotes complex conjugate) and integrating it with respect to y over $(-\infty, \infty)$, we obtain, after integration by parts and use of the boundary condition (12),

$$I_1 c = -I_2,$$

where

$$I_1 = \int_{-\infty}^{\infty} \frac{\beta}{\beta + H_y} H_y |\phi|^2 \, dy,$$

$$I_2 = \int_{-\infty}^{\infty} \left[\frac{\beta}{\beta + H_y} P_y H_y |\phi|^2 + H(H_y)^2 (|\phi_y|^2 + k^2 |\phi|^2) + \beta H H_y |\phi|^2 \right] dy.$$

Since $\operatorname{Im} I_1 = \operatorname{Im} I_2 = 0$, the condition

$$I_1 \neq 0 \tag{19}$$

guarantees that $c = -I_2/I_1$ is real, which can be achieved by assuming that

$$H_y(y) \geqslant 0 \tag{20a}$$

or

$$-\beta \leqslant H_y(y) \leqslant 0 \tag{20b}$$

or

$$H_y(y) \leqslant -\beta. \tag{20c}$$

Thus, flows that satisfy any of conditions (20a, b, c) are stable. From a physical viewpoint (20a) guarantees the stability of an arbitrary westward flow, while (20b) is more restrictive and guarantees the stability of only weak eastward flows. Indeed, for the parameter values representative of the subarctic front in the North Pacific (Roden, 1975), i.e. $H = 5.5\,\mathrm{km}$, $\Delta\rho/\rho = 1.3 \times 10^{-3}$ and $\theta = 41°30'$, criterion (9b) becomes

$$0 \leqslant \tilde{v} \lesssim 13\,\mathrm{cm/s},$$

where $\tilde{v}$ is the dimensional velocity in the upper layer. Finally, condition (20c) describes a flow that does not decay at $y \to \pm\infty$.

It should be emphasized that (20) is only a sufficient criterion of stability: flows that do not satisfy it are not necessarily unstable. However, numerical simulation of fronts with thin upper layer (Pavia, 1992; Benilov, 1993), as well as some analytical arguments (Benilov, 1993), show that double fronts (H_y changes sign) are unstable. We also have numerical and analytical evidence (Benilov, 1993) of instability of strong eastwards flows violating conditions (20b).

Finally, we note that in those strong-β-effect cases, where the instability does occur (Benilov, 1993), it has nothing to do with the weak-β-effect instability: in the transitional regime the latter shifts towards the short-wave region and eventually disappears from the asymptotic governing equations (7) completely.

4. A WEDGE-LIKE FRONT

In this section we shall consider the wedge-like front [see Cushman-Roisin (1986)] characterized by a uniform flow in the upper layer and a uniformly sheared flows in the lower layer:

$$H(y)=\begin{cases}-vy & \text{for} \quad vy\leqslant 0,\\ 0 & \text{for} \quad vy>0;\end{cases} \qquad P(y)=\begin{cases}-uy+\frac{1}{2}sy^2 & \text{for} \quad vy\leqslant 0,\\ 0 & \text{for} \quad vy>0.\end{cases} \tag{21}$$

This flow always meets one of conditions (20) and, according to the above stability analysis, is stable. This simple example can provide a realistic estimate of the time scale of stable perturbations propagating along oceanic fronts. Substitution of (21) into (18) leads to the following equation for ϕ:

$$y\phi_{yy}+\phi_y-(Av^2y+c')\phi=0, \tag{22}$$

where

$$\left.\begin{aligned} A&=\frac{s\beta}{v^4(\beta-v)}+\frac{k^2}{v^2}-\frac{\beta}{v^3},\\ c'&=\frac{\beta(c-u)}{v^2(\beta-v)}. \end{aligned}\right\} \tag{23}$$

Equation (22) has a singularity at $y=0$ (where the front intersects the surface of the ocean). We shall therefore assume that

$$|\phi|<\infty \qquad \text{at} \qquad y=0. \tag{24a}$$

The other boundary condition results from our restriction to disturbances localized near the flow:

$$\phi\to 0 \qquad \text{as} \qquad (vy)\to-\infty. \tag{24b}$$

The solution to (22), (24) strongly depends on the sign of A. The case $A<0$ corresponds to a continuous spectrum of non-trapped waves [$\phi(y)$ oscillates as $(vy)\to-\infty$]. These waves are long Rossby waves modified by the vertically and horizontally sheared current.

If $A>0$, the substitution

$$\phi(y)=\psi(z)e^{-z/2}, \qquad z=-2v\sqrt{A}\,y$$

reduces (22) to the Laguerre equation:

$$z\psi_{zz}+(1-z)\psi_z+\left(\frac{c'}{2v\sqrt{A}}-\frac{1}{2}\right)\psi=0,$$

whose solution is bounded only if

$$\frac{c'}{2v\sqrt{A}} - \frac{1}{2} = n,$$

where $n \geqslant 0$ is an integer. Taking into account (23) to return to the earlier notation, we obtain the phase speed of the wave perturbations:

$$c_n = u + (2n+1)\frac{v(\beta - v)}{\beta}\sqrt{k^2 v^2 + \frac{s\beta}{\beta - v} - \beta v}. \tag{25}$$

It is worth noting that the sign of the second term in formula (25) may be opposite to that of v. As a result, disturbances may propagate upstream.

Finally, assuming for simplicity that $u = s = 0$ (no flow in the bottom layer), we shall evaluate c_n for the subarctic front in the North Pacific (Roden, 1975). We assume that $H = 5.5\,\text{km}$, $\Delta\rho/\rho = 1.3 \times 10^{-3}$, $\theta = 41^\circ 30'$ and the (dimensional) velocity in the upper layer is $\tilde{v} = 20\,\text{cm/s}$ (which corresponds to a frontal flow with $H_1 = 300\,\text{m}$ and $L = 200\,\text{km}$). For the disturbance with wavelength 100 km, formula (25) yields

$$\tilde{c}_o \approx 1.4\,\text{cm/s}, \qquad \tilde{c}_1 \approx 4.2\,\text{cm/s}, \quad \cdots$$

which indicates that nearly geostrophic disturbances on eastward zonal oceanic fronts propagate very slowly. Remarkably, stable disturbance on the westward flow with the same parameters propagate considerably faster:

$$\tilde{c}_o \approx 6.6\,\text{cm/s}, \qquad \tilde{c}_1 \approx 19.8\,\text{cm/s}, \quad \cdots$$

Several dispersion curves (25) are shown in Figure 2.

5. SHORT-WAVE INSTABILITY OF FLOWS WITH STRONG β-EFFECT

It should be noted, however, that use of scaled equations like (4) in a stability analysis is always subject to criticism that possible instabilities have been "scaled out" of the problem. Accordingly, we should examine the stability of strong-β-effect flows with respect to *short* perturbations, which are not described by our asymptotic equations.

Generally speaking, in order to take into account short disturbances, we should include the terms $\sim \Delta h_t$ into the first equation of the original system (4) [see Cushman-Roisin *et al.*, 1992, equations (25), (26)], and then follow the standard scheme of stability analysis. It turns out, however, that the short-wave stability of geostrophic flows can be examined in a much simpler way.

First of all, we observe that, if the wavelength of the disturbance is much smaller than the effective spatial scale of the mean flow, the stability analysis can be carried out *locally* in the approximation of small-amplitude geostrophic flows (indeed, mean variations of the upper-layer depth over the wavelength of a short perturbation is much

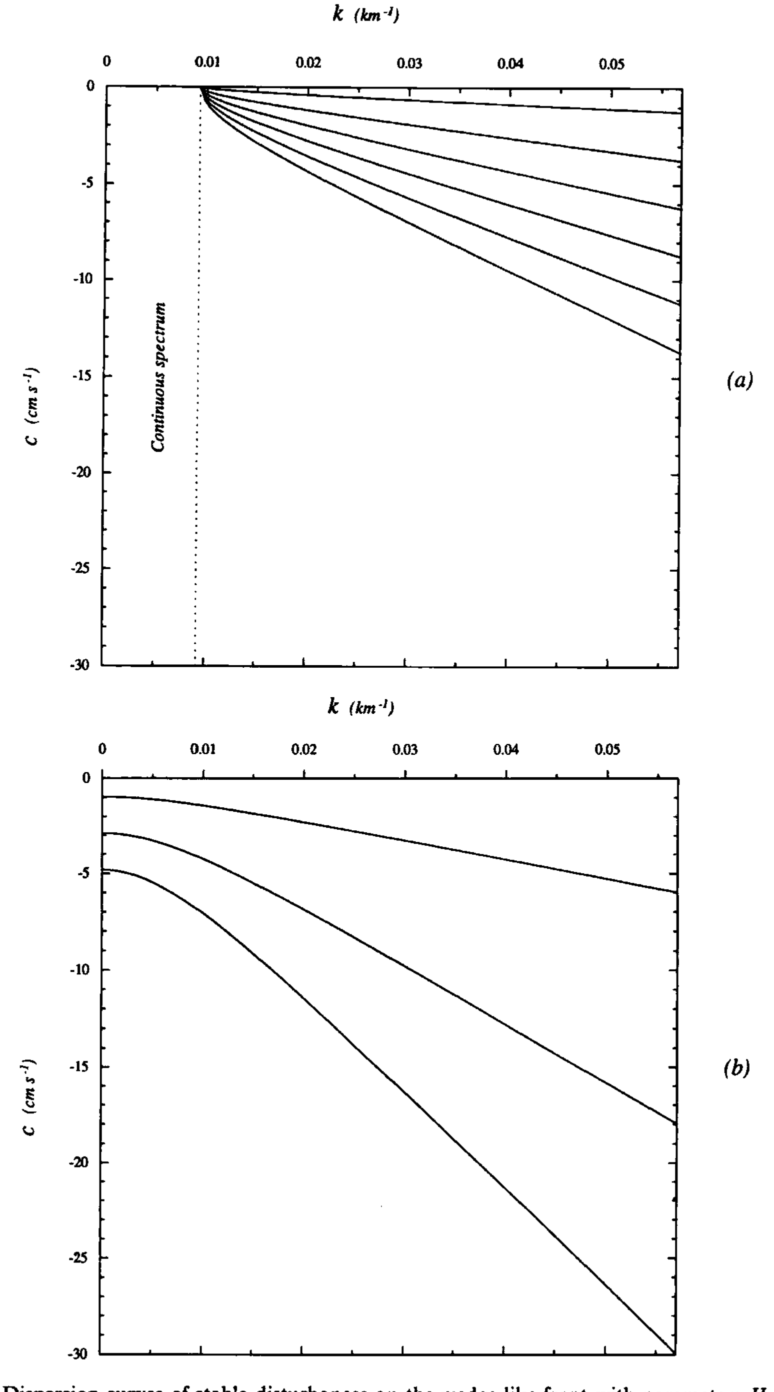

Figure 2 Dispersion curves of stable disturbances on the wedge-like front with parameters $H = 5.5\,\text{km}$, $\Delta\rho/\rho = 1.3 \times 10^{-3}$, $\theta = 41°30'$, $|\tilde{v}| = 20\,\text{cm/s}$, $s = u = 0$ (which corresponds to a frontal flow with $H_1 = 300\,\text{m}$ and $L = 200\,\text{km}$). a) eastward flow ($\tilde{v} > 0$), $n = 0$–5. b) westward flow ($\tilde{v} < 0$), $n = 0$–2.

smaller than its local value). Accordingly, we can make use of the results of Phillips' model (1954). Writing the stability criterion of the latter in the non-dimensional form

$$|U_s| \leqslant \begin{cases} \beta H & \text{if} \quad U_s < 0, \\ \beta(1-H) & \text{if} \quad U_s > 0; \end{cases}$$

where $U_s = -H_y$ is the non-dimensional shear velocity; we see that the flow is stable with respect to short disturbances only if

$$\beta H \geqslant H_y \geqslant -\beta(1-H). \tag{26}$$

In the thin-upper-layer limit, criterion (26) can be reduced to

$$0 \gtrsim H_y \gtrsim -\beta$$

and coincides with condition (20b). Correspondingly, flows (20b) are stable with respect to all disturbances, whereas flows (20a, c) are stable only with respect to long and "medium" disturbances and (locally) unstable with respect to short perturbations.

It can be conjectured that short-wave and "medium-wave" instabilities entail different behaviours of the mean flow. For one thing, the former is unlikely to destroy the flow, as it usually leads to randomization of unstable disturbances, and the resulting turbulent friction may stabilize the flow. Eventually, the short-wave instability may saturate at some level, while the "medium-wave" instability results in meandering of the mean flow and can break it up completely.

6. CONCLUSIONS

Thus, we have considered the stability of two-layered frontal flows with thin upper layer. It has been demonstrated that, if the β-effect is weak:

$$\beta \lesssim \varepsilon^{3/2},$$

all fronts are unstable. In the transitional regime:

$$\varepsilon^{3/2} \lesssim \beta \lesssim \varepsilon$$

the instability shifts towards the short-wave region and in the regime of strong β-effect:

$$\beta \gtrsim \varepsilon$$

may disappear completely [specifically, the short-wave instability disappears for moderate westward flows that satisfy condition (20b)]. However, the regime of strong β-effect has its own set of instabilities (with respect to disturbances whose wavelength is of the order of the width of the flow), and the "medium-wave" stability can be guaranteed only for frontal flows that satisfy one of conditions (20a, b, c). Summary of the results of this paper and Benilov (1992a) can be given in the following table:

	Weak β-effect: $\beta \sim \varepsilon^{3/2}$	*Strong β-effect*: $\beta \sim \varepsilon$
$h \sim 1$	Benilov (1992a) instability	Benilov (1992a) stability
$h \sim \varepsilon$	this paper instability	this paper stability/instability

Finally, we should mention that the above analysis can be generalized for flows over weak ($\delta H \ll H$) bottom topography.

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