1. Consider the affine control system

$$\dot{\mathbf{x}} = \begin{pmatrix} x_1 + x_2^2 \\ x_1 + x_2^3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u.$$

$$y = x_2$$

   (a) Find the relative degree for the given output. Show that it equals the system order and find the normal form of the system equations.

   (b) Find the I/O linearisation feedback of the normal form system.

   (c) Find the controller that places the poles of this linear system at -1, -2.

   (d) Linearise the original system about the fixed point $\mathbf{x}_e = \mathbf{0}$, and construct the linear feedback controller that places the poles at -1, -2.

2. Consider the system with the same state equation as Q1, but with the output

$$y = x_1$$

   (a) Find the relative degree for the given output. Show that it is less than the system order, and find the normal form of the system.

   (b) Find the I/O linearisation feedback of the normal form system.

   (c) Investigate the zero dynamics of the "remnant"; show that it is equivalent to studying the dynamics of

$$\dot{\eta}_2 = C'(C^{-1}(\eta_2))(C^{-1}(\eta_2))^3$$

   where $C(.)$ is an invertible function with derivative $C'(.)$ defined in a neighbourhood of $\eta_e = 0$. Can anything be said be said in general about the stability of the fixed point $\eta_e = 0$ and what does this say about using I/O linearisation feedback in this case?

3. Consider the system with the same state equation as Q1, but with no output. Find the state transformation and feedback control that brings about the input-state linearisation of the system.