MS4328 Problem Sheet 6

1. A square matrix A is said to be a "stability matrix" if all its eigenvalues have real part less than 0. It is said to be a "convergent matrix" if all its eigenvalues have modulus less than 1.

   (a) Solve the *Lyapunov* Equation to check whether the matrices given below are stability matrices or otherwise

$$\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \qquad \begin{pmatrix} -1 & 2 \\ -0.5 & 0.5 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}$$

   (b) Solve the Discrete *Lyapunov* Equation to check whether the matrices given below are convergent matrices or otherwise

$$\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \qquad \begin{pmatrix} -1 & 2 \\ -0.5 & 0.5 \end{pmatrix}$$

2. By using the approach of "Specified Control Structure" or otherwise show how to choose a control strategy that stabilises the following systems

   (a)
$$\ddot{x} - \dot{x} + x^2 = u$$

   (b)
$$\ddot{x} + \beta\dot{x}^3 + \alpha x = u$$
   where it is known that $|\beta| < 2$ and $4 < \alpha < 6$.

   (c)
$$\ddot{x} + \dot{x}^5 = x^2 u$$

   (d)
$$\ddot{x} + \beta x\dot{x} + \alpha x^2 = u$$
   where it is known that $-1 < \beta < 2$ and $|\alpha| < 3$.

3. (a) Show that $V(\mathbf{x}) = \frac{1}{2}x_1^2 + x_1x_2 + \frac{3}{2}x_2^2$ is a control-*Lyapunov* function for the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2) + u \end{aligned}$$

   where $f(.,.)$ is any function such that $f(0,0) = 0$.

   (b) Show that $V(\mathbf{x}) = \frac{1}{2}x_1^2 + x_1x_2 + x_2^2$ is a control-*Lyapunov* function for the system

$$\begin{aligned} \dot{x}_1 &= x_2 + 4x_1^2x_2 \\ \dot{x}_2 &= -x_1 + u \end{aligned}$$

   Find a control that stabilizes the system. What does *Sontag*'s formula say?

   (c) Show that $V(\mathbf{x}) = x_1^2 + 2x_2^2$ is a control-*Lyapunov* function for the system

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= x_1^2(k) + u(k) \end{aligned}$$