All questions deal with continuous-time LTI systems.

1. Which of the following systems can be decoupled by linear state feedback?

   (a)
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & 3 \\ 0 & 0 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & -2 \\ 3 & 7 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   (b)
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 2 \end{pmatrix}, \qquad C = \begin{pmatrix} 3 & 6 & 1 \\ 2 & 0 & 0 \end{pmatrix}$$

   (c)
$$A = \begin{pmatrix} 0 & 1 \\ 4 & -3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$$

2. Of the systems that are decouplable by linear state feedback in Q1,

   (a) Find the feedback that "integrator-decouples the system".

   (b) Is the remnant stable?

   (c) Find the additional linear feedback that places all integrator decoupled poles at -2.

3. Consider the problem of designing a linear state feedback that decouples the square MIMO system described by

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} &= C\mathbf{x} + D\mathbf{u} \end{aligned}$$

   where the $p \times p$ matrix D may or may not have zero rows.
   How does the definition of decoupling index change and how does this in turn influence the development of the resultant theory (See Eq(4) in Notes 10).
   Illustrate these ideas with the system given by

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & 3 \\ 0 & 0 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & -2 \\ 3 & 7 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$