MS4328 Problem Sheet 3

1. Is the following system completely controllable? If not identify which modes are not controllable.

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & 0 \\ 2 & 2 \\ 1 & 2 \end{pmatrix} \mathbf{u}, \qquad \mathbf{y} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & -1 \end{pmatrix} \mathbf{x}$$

Is the system stabilisable ?

2. Which of the following single input linear systems described by (A, b) pairs are completely controllable?

(a)
$$A = \begin{pmatrix} -23 & 40 \\ -15 & 26 \end{pmatrix}, \qquad b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

(b)
$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(c)
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}, \qquad b = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

For those that are, find the linear transformation that brings them to controllable canonical form, and write down the transformed equations.

3. A system is null Ω-controllable if it is null controllable for $u \in \Omega$. Find the set of null Ω-controllable states for the following:

(a)
$$x_{k+1} = 2x_k + u_k, \qquad | u_k | \leq 4$$

(b)
$$x_{k+1} = 2x_k + u_k, \qquad -2 \leq u_k \leq -1$$

(c)
$$\mathbf{x}_{k+1} = \begin{pmatrix} 0 & 1 \\ -1 & \frac{5}{2} \end{pmatrix} \mathbf{x}_k + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_k, \qquad | u_k | \leq 1$$

Hint: Find the set of states S_1 that can be brought to $\mathbf{0}$ in 1 time step; then find the set of new states S_2 that can be brought to S_1 in 1 time step. Repeat this process recursively.

If there were no constraints on u, which of the above would be null controllable?