1. Consider the following matrices:

$$A_1 = \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 1 \\ -4 & -4 \end{pmatrix}, \qquad A_4 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix}, \qquad A_6 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

   Which of them are (i) diagonalisable (ii) in Jordan Form ?
   Compute A_i^k, $k \in \mathbf{N}_0$ and $e^{A_i t}$, $t \in \mathbf{R}$ for each of the matrices.

2. Find solutions for the following equations:

   (a)

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \qquad \mathbf{x}(0) = \mathbf{0}, \;\; u(t) = 1 - e^{-2t}, t \geq 0$$

   (b)

$$\mathbf{x}_{k+1} = \begin{pmatrix} 0 & 1 \\ -4 & -4 \end{pmatrix} \mathbf{x}_k + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_k \qquad \mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \;\; u_k \equiv 1, k \geq 0$$

   (c)

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \;\; u(t) = 3t, t \geq 0$$