1. It's possible to model this circuit as 3rd order. However as the voltage drop (v) across the series connection of the two capacitors determines the drops across the individual capacitors (at least for both initially uncharged), it is reasonable to combine the two into an equivalent capacitor with capacitance C where $1/C = 1/C_1 + 1/C_2$, and model the system as second order.
   Making the natural choice for state variables - the variables associated with energy storage - we get $\mathbf{x} = (i_L,\, v)^T$, and the state model is

$$\begin{aligned} \frac{d}{dt} i_L \left(= \frac{R i_R + v}{L} \right) &= \frac{R(i - i_L) + v}{L} \\ \frac{d}{dt} v \left(= \frac{i_R}{C} \right) &= \frac{i - i_L}{C} \\ \text{output} \qquad v &= v \qquad \text{state variable} \end{aligned}$$

   or, since it is a linear system, it can be written in matrix-vector form as

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} i_L \\ v \end{pmatrix} &= \begin{pmatrix} -\frac{R}{L} & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{pmatrix} \begin{pmatrix} i_L \\ v \end{pmatrix} + \begin{pmatrix} \frac{R}{L} \\ \frac{1}{C} \end{pmatrix} i \\ v &= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} i_L \\ v \end{pmatrix} \end{aligned}$$

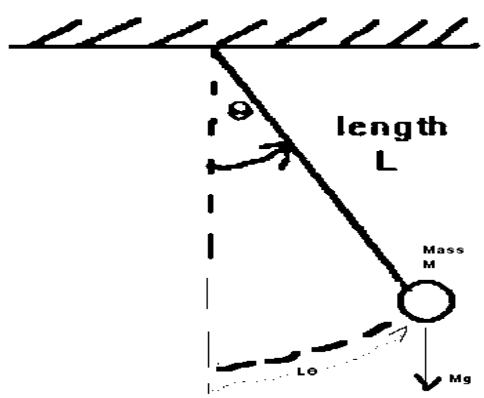

Figure 1: Diagram for Q2

2. This mechanical system is 2nd order, and does not have an external driving force (or control, in the context of this module). The force acting on the pendulum bob

causing it to move is the component of the weight acting in the direction of motion but in the opposite direction to increasing θ, i.e. $-mg\sin\theta$. [The other component ($mg\cos\theta$) is balanced by the tension in the pendulum arm.] *Newton*'s 2nd law of motion then gives

$$
\begin{aligned}
\frac{d}{dt}mv &= F \\
\Rightarrow \frac{d}{dt}(m\frac{d}{dt}(L\theta)) &= -mg\sin\theta \\
\Rightarrow \frac{d^2\theta}{dt^2} &= -\frac{g}{L}\sin\theta
\end{aligned}
$$

With the choice $\mathbf{x} = (x_1,\ x_2) \triangleq (\theta,\ d\theta/dt)^T$, the state model is

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{g}{L}\sin x_1 \\
\theta &= x_1
\end{aligned}
$$

The system is in equilibrium when the pendulum is at rest hanging vertically downwards, i.e. $\mathbf{x}_e = (0,0)^T$. (Is there another equilibrium ?) With $\delta\mathbf{x} = \mathbf{x}$, and noting that here $\sin\theta \approx \theta$ the linearised model is

$$
\begin{aligned}
\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
\theta &= \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}
\end{aligned}
$$

3. (a) Order =2. With choice $\mathbf{x} = (h_1,\ h_2)^T$, the state model is

$$
\begin{aligned}
\dot{h}_1 \left(= \frac{\dot{V}_1}{a_1} = \frac{u-q}{a_1}\right) &= \frac{u - c_1\sqrt{h_1}}{a_1} \\
\dot{h}_2 \left(= \frac{\dot{V}_2}{a_2} = \frac{q-y}{a_2}\right) &= \frac{c_1\sqrt{h_1} - c_2\sqrt{h_2}}{a_2} \\
y &= c_2\sqrt{h_2}
\end{aligned}
$$

(b) In steady state, $u = q = y = \ldots c_2\sqrt{H}$.
Note that in steady state

$$
h_1 \triangleq H_1 = \left(\frac{c_2}{c_1}\right)^2 H.
$$

(c) With $u = c_2\sqrt{H} + \delta u,\ h_1 = H_1 + \delta h_1,\ h_2 = H + \delta h_2$ and $y = c_2\sqrt{H} + \delta y$, the linearised model is

$$
\begin{aligned}
\frac{d}{dt}\begin{pmatrix} \delta h_1 \\ \delta h_2 \end{pmatrix} &= \begin{pmatrix} -\frac{c_1}{2a_1\sqrt{H_1}} & 0 \\ \frac{c_1}{2a_1\sqrt{H_1}} & -\frac{c_2}{2a_2\sqrt{H}} \end{pmatrix}\begin{pmatrix} \delta h_1 \\ \delta h_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{a_1} \\ 0 \end{pmatrix}\delta u \\
\delta y &= \begin{pmatrix} 0 & \frac{c_2}{2\sqrt{H}} \end{pmatrix}\begin{pmatrix} \delta h_1 \\ \delta h_2 \end{pmatrix}
\end{aligned}
$$