1. When $f(x) = x(1-x)$, $f'(x) = 1-2x$. Also $x_e = 0, 1$, thus

$$\begin{aligned} f'(0) = 1 &\Rightarrow \text{ unstable} \\ f'(1) = -1 &\Rightarrow \text{ locally asymptotically stable} \end{aligned}$$

When $f(x) = -3x$, $f'(x) = -3$. Now $x_e = 0$, thus

$$f'(0) = -3 \Rightarrow \text{ locally asymptotically stable}$$

When $f(x) = x^2(1-x)$, $f'(x) = 2x - 3x^2$. Now $x_e = 0, 1$ giving

$$\begin{aligned} f'(0) = 0 &\Rightarrow \text{ ? \quad unstable (graphically)} \\ f'(1) = -1 &\Rightarrow \text{ locally asymptotically stable} \end{aligned}$$

2. When $f(x) = rx(1-\frac{x}{K})$, $f'(x) = r(1-2\frac{x}{K})$. Also $x_e = 0, K(1-\frac{1}{r})$, leading to

$$\begin{aligned} f'(0) = r &\Rightarrow \text{ locally stable if } |r| < 1 \\ f'(K(1-\frac{1}{r})) = 2-r &\Rightarrow \text{ locally stable if } 1 < r < 3 \end{aligned}$$

When $f(x) = 2x$, $f'(x) = 2$. Also $x_e = 0$, thus

$$f'(0) = 2 \Rightarrow \text{ unstable}$$

When $f(x) = \frac{1}{2}x^2(1-x)$, $f'(x) = x - \frac{3}{2}x^2$. Also $x_e = 0$, thus

$$f'(0) = 0 \Rightarrow \text{ locally `superstable'}$$

When $f(x) = 5x^2(1-x)$, $f'(x) = 10x - 15x^2$. And $x_e = 0, \frac{1}{2} \pm \frac{1}{2\sqrt{5}}$, leading to

$$\begin{aligned} f'(0) = 0 &\Rightarrow \text{ locally `superstable'} \\ f'(\frac{1}{2} - \frac{1}{2\sqrt{5}}) = \frac{1+\sqrt{5}}{2} &\Rightarrow \text{ unstable} \\ f'(\frac{1}{2} + \frac{1}{2\sqrt{5}}) = \frac{1-\sqrt{5}}{2} &\Rightarrow \text{ locally stable} \end{aligned}$$

3. The system may be rewritten

$$\dot{\mathbf{x}} = \begin{pmatrix} -1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Solving $\mathbf{0} = \begin{pmatrix} -1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{x}_e + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ yields

$$\mathbf{x}_e = \begin{pmatrix} -0.3 \\ -0.1 \end{pmatrix}$$

Furthermore the Jacobian matrix is $\begin{pmatrix} -1 & 3 \\ 3 & 1 \end{pmatrix}$ which has eigenvalues $\pm\sqrt{10}$. There is an eigenvalue with positive real part, hence $\mathbf{x}_e$ is unstable.

4. Solving $x_e = 3x_e(1 - x_e - y_e), \;\; y_e = 4y_e(1 - y_e - \frac{3}{2}x_e)$ yields fixed points

$$\mathbf{x}_e = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 \\ 3/4 \end{pmatrix}, \qquad \begin{pmatrix} 2/3 \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} 1/6 \\ 1/2 \end{pmatrix}$$

The Jacobian matrix is

$$\begin{pmatrix} 3 - 6x - 3y & -3x \\ -6y & 4 - 8y - 6x \end{pmatrix}$$

So

Fixed Point	Jacobian	Eigenvalues	Stable ?
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$	3, 4	No
$\begin{pmatrix} 0 \\ 3/4 \end{pmatrix}$	$\begin{pmatrix} 3/4 & 0 \\ -9/2 & -2 \end{pmatrix}$	3/4, -2	No
$\begin{pmatrix} 2/3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix}$	-1,0	?
$\begin{pmatrix} 1/6 \\ 1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 & -1/2 \\ -3 & -1 \end{pmatrix}$	$-1/4 \pm \sqrt{33}/4$	No