Mathematical Induction

Mathematical Induction (MI) is a proof method used to establish the truth of a statement P about the natural number n. Examples of such statements are:

- $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
- The number of subsets of a set with n elements is 2^n

The steps involved in MI are

1. Base Step : Establish P for some initial value $n = n_0$
2. Inductive Step: Show that if P is true for $n = k$, then P is also true for $n = k + 1$.

The combination of these two steps ensures that P is true for all n greater than or equal to n_0.(why ?)

Example: Use MI to prove P : $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
Base Step: When $n = 1$ the left hand side (LHS) of P is 1. The RHS is $\frac{1(1+1)}{2} = 1$.
Inductive Step: Assume P is true for $n = k$ i.e.

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$$

When $n = k + 1$, the LHS of P is

$$1 + 2 + 3 + \cdots + k + (k + 1) = [1 + 2 + 3 + \cdots + k] + (k + 1)$$

And using the inductive assumption, this becomes

$$\frac{k(k+1)}{2} + (k + 1) = (k + 1)(\frac{k}{2} + 1) = \frac{(k + 1)(k + 2)}{2}$$

which is the RHS of P when $n = k + 1$. This completes the proof, and shows that the result is true for $n \geq 1$.

The version of MI given in the box above is called *weak* induction. *Strong* induction consists of

1. Base Step : Establish P for some initial value $n = n_0$
2. Inductive Step: Show that if P is true for $n = 1, 2, 3, \ldots, k$, then P is also true for $n = k + 1$.

Both versions are equivalent.