MA4006: Exercise Sheet 3

Please attempt these questions before the tutorials in Weeks 4 and 5.

1. Evaluate the integral $\iint_R dx\,dy$ over the triangle with vertices $(-1,0)$, $(0,2)$ and $(2,0)$.

2. Evaluate the integral $\iint_R y^{-1/2}\,dx\,dy$ over the area bounded by $y=x^2$, $x+y=2$, and the y-axis.

3. Evaluate $\int_C \Omega\,ds$ where $\Omega(x,y)=x^2+y^2$ and C is the segment of the line $y=3x$ from $(0,0)$ to $(2,6)$.

4. Evaluate $\int_C f\,ds$ where $f(x,y,z)=1+y^2+z^2$ and $C:\mathbf{r}=(t,\cos t,\sin t)$, $0\leq t\leq 2\pi$.

5. Find the work done in moving a particle in a force field given by

$$\mathbf{F}=3xy\mathbf{i}-y^2\mathbf{j}+10x\mathbf{k}$$

along the curve $y=2x^2$, $z=0$, from the point $(0,0,0)$ to the point $(1,2,0)$.

6. If $\mathbf{f}=8x^2yz\mathbf{i}+5z\mathbf{j}-4xy\mathbf{k}$, find the work done in moving a particle in a force field given by $\mathbf{f}$ along the curve C with parametric definition $\mathbf{r}(t)=(t,t^2,t^3)$, $(0\leq t\leq 1)$.

7. Show that $\mathbf{f}=y^2\mathbf{i}+2xy\mathbf{j}$ is a conservative vector field. Determine and associated *scalar potential* ϕ for this vector field $\mathbf{f}$.

8. If $\mathbf{a}=(3x^2+6y,-14yz,20xz^2)$ evaluate $\int_C \mathbf{a}\cdot d\mathbf{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths:

(a) The straight lines from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and finally to $(1,1,1)$.

(b) The straight line from $(0,0,0)$ to $(1,1,1)$.

9. Let $\phi=4x$ and let V denote the closed region bounded by the planes $4x+2y+z=8$, $x=0$, $y=0$ and $z=0$. Evaluate $\iiint_V \phi\,dV$.

10. Using Green's Theorem, evaluate the line integral $\oint_C \mathbf{F}\cdot d\mathbf{r}$ counterclockwise around the boundary of C of the region R, where $\mathbf{f}=(\sin y,\cos x)$ and R is the triangle with vertices $(0,0)$, $(\pi,0)$ and $(\pi,1)$.

11. Evaluate $\iint_S \mathbf{a}\cdot\hat{\mathbf{n}}\,dS\equiv\iint_S \mathbf{a}\cdot d\mathbf{S}$ where $\mathbf{a}=(18z,-12,3y)$ and S is part of the plane $2x+3y+6z=12$ located in the first octant.

12. Use the Divergence Theorem to compute the surface integral $\iint_S \mathbf{a} \cdot \mathbf{dS}$ where $\mathbf{a} = (4x, -2y^2, z^2)$ and S is the cylindrical surface $x^2 + y^2 = 4$, $0 \leq z \leq 3$.

13. Verify Stokes theorem for the case $\mathbf{f} = (2x - y, -yz^2, -y^2 z)$ where the surface S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$ and the curve C is defined as the unit circle.