1. Use *Gaussian* Elimination to solve

$$\begin{pmatrix} 4 & -2 & 5 \\ 2 & 6 & -3 \\ 1 & 7 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix}$$

2. Find a LU factorisation of

$$A = \begin{pmatrix} 2 & 4 & 4 \\ 1 & 0 & 3 \\ 2 & -7 & 5 \end{pmatrix}$$

Use the factorisation to consecutively solve the systems $Ax_i = b_i,\ i = 1, 2$ where

$$b_1 = \begin{pmatrix} 2 \\ -4 \\ -21 \end{pmatrix} \qquad b_2 = x_1$$

3. Find the *Cholesky* decomposition of the matrix

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 4 & -2 \\ 1 & -2 & 3 \end{pmatrix}$$

and hence solve $Ax = b$ where

$$b = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$$

4. The condition number of a matrix A is defined as $\kappa(A) = ||A||\ ||A^{-1}||$.

   (a) Compute $\kappa(A)$ exactly when

$$A = \begin{pmatrix} 0.2 & 0.01 \\ 4 & 0.1 \end{pmatrix}$$

   (b) Estimate $\kappa(A)$ where A is as in Question (2).

   (c) Estimate $\kappa(A)$ where A is as in Question (3).

5. (a) By considering the eigenvalues of the iteration matrix, show that the *Jacobi* method is guaranteed to converge for the system of linear equations

$$\begin{pmatrix} 1 & 1 & -8/13 \\ 8/13 & 1 & 8/13 \\ 8/13 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -22 \\ 12 \\ 9 \end{pmatrix}$$

   (b) Use 4 iterations of the method, with initial guess $x_1(0) = 0,\ x_2(0) = 0,\ x_3(0) = 0$, to obtain a numerical solution of the system.

   (c) Is the *Gauss-Seidel* method guaranteed to converge for this system?