

1. Which of the following are subspaces of the space of real valued functions of a real variable:
  - (a) all functions  $f$  such that  $f(0) = 0$ ,
  - (b) all functions  $f$  such that  $f(x) \leq 0$  for all  $x$ ,
  - (c) all polynomials of degree at most 3 with integer coefficients.
2. Let  $S$  be the following set of vectors (over  $\mathbf{R}^4$ ):

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 2 \\ -4 \\ 6 \end{pmatrix}, \begin{pmatrix} -1 \\ 10 \\ 8 \\ 6 \end{pmatrix} \right\}$$

- (a) Is  $S$  linearly independent ?
  - (b) Can the vector  $(7, 2, 10, -6)^T$  be expressed as a *linear combination* of the vectors of  $S$  ?
3.  $P_n$  is the space of polynomials of degree at most  $n$  over a real variable.
  - (a) What is the dimension of  $P_n$ ?
  - (b) Show that  $\{1, x, x^2, \dots, x^n\}$  is a basis for  $P_n$ .
  - (c) Show that  $\{1+x, 2+x, 2-x^2\}$  is a basis for  $P_2$  and write the polynomial  $x^2+1$  in terms of this basis.
4. (a) What is the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 1 & 4 \\ 1 & 5 & 7 & 0 \\ 0 & 0 & 3 & -2 \\ 1 & 0 & 1 & 4 \end{pmatrix}$$

- (b) Solve the system of equations

$$\begin{pmatrix} 1 & 2 & 1 & 4 \\ 1 & 5 & 7 & 0 \\ 0 & 0 & 3 & -2 \\ 1 & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 17 \\ -24 \\ -19 \\ 19 \end{pmatrix}$$