

1. Solve the following initial value problems by means of *Laplace* transforms:

(a)

$$\frac{dy}{dt} + y = t - 3, \quad t \geq 0; \quad y(0) = 2$$

(b)

$$\frac{dy}{dt} - 4y = 3 \sin 2t, \quad t \geq 0; \quad y(0) = 0$$

(c)

$$\frac{dy}{dt} + 3y = 6u_1(t), \quad t \geq 0; \quad y(0) = 1$$

(d)

$$\frac{dy}{dt} + 2y = f(t) = \begin{cases} 1, & \text{if } 0 \leq t < 1 \\ -1, & \text{if } 1 \leq t < 2 \\ 0, & \text{if } 2 \leq t \end{cases}; \quad y(0) = 1$$

(e)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = t + 2, \quad t \geq 0; \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 1$$

(f)

$$\frac{d^2y}{dt^2} + 4y = e^{2t}, \quad t \geq 0; \quad y(0) = 2, \quad \frac{dy}{dt}(0) = 0$$

(g)

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^{-t}, \quad t \geq 0; \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 0$$

(h)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 4u_0(t), \quad t \geq 0; \quad y(0) = 2, \quad \frac{dy}{dt}(0) = 0$$

2. Compute the following convolutions $f * g$ by definition or by *Laplace* Transform:

(a)

$$f(t) = 1, \quad g(t) = \sin 2t$$

(b)

$$f(t) = e^{-t}, \quad g(t) = 1$$

(c)

$$f(t) = t, \quad g(t) = t$$

(d)

$$f(t) = t, \quad g(t) = e^{2t}$$

3. Solve the following initial value problem by convolution

$$\frac{dy}{dt} + 3y = \cos 4t, \quad t \geq 0; \quad y(0) = 0$$

4. Solve the integral equation

$$y(t) = \sin t + 2 \int_0^t \cos(t-u)y(u) du$$