UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

**MID TERM ASSESSMENT PAPER**

MODULE CODE: MA4003

SEMESTER: Autumn 2010/11

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: 45 minutes

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 20 %

Colour: Yellow

INSTRUCTIONS TO CANDIDATES: **Answer all questions. All questions carry equal marks. Use the Answer Sheet below.**

ANSWER SHEET

STUDENT'S NAME:
STUDENT'S ID NUMBER:

For each question, place an "**X**" in the box of your choice.

| Question | a | b | c | d | e | Do not write in this column |
|---|---|---|---|---|---|---|
| 1 | | | | **X** | | |
| 2 | **X** | | | | | |
| 3 | | | **X** | | | |
| 4 | | | | | **X** | |
| 5 | | | | **X** | | |
| 6 | | | | | **X** | |
| 7 | | | | **X** | | |
| 8 | | | | **X** | | |
| 9 | | | **X** | | | |
| 10 | | | **X** | | | |

**Table of Laplace Transforms**

| $f(t),\ t \geq 0$ | $F(s) = \mathcal{L}[f(t)]$ |
|---|---|
| $1$ | $\frac{1}{s}$ |
| $t$ | $\frac{1}{s^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ |
| $\sinh at$ | $\frac{a}{s^2-a^2}$ |
| $\cosh at$ | $\frac{s}{s^2-a^2}$ |
| $\frac{1}{a-b}(e^{at} - e^{bt})$ | $\frac{1}{(s-a)(s-b)}$ |
| $\frac{a}{a-b}e^{at} - \frac{b}{a-b}e^{bt}$ | $\frac{s}{(s-a)(s-b)}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ |
| $f'(t)$ | $sF(s) - f(0)$ |
| $f''(t)$ | $s^2F(s) - sf(0) - f'(0)$ |
| $\int_0^t f(\tau)\, d\tau$ | $\frac{1}{s}F(s)$ |
| $e^{at}f(t)$ | $F(s-a)$ |
| Heaviside $u_a(t)$ | $\frac{e^{-as}}{s}$ |
| $f(t-a)u_a(t)$ | $e^{-as}F(s)$ |
| Ramp $R(t-a)$ | $\frac{e^{-as}}{s^2}$ |
| $tf(t)$ | $-F'(s)$ |
| $\dfrac{f(t)}{t}$ | $\displaystyle\int_s^\infty F(\sigma)\, d\sigma$ |
| $(f * g)(t) \equiv \displaystyle\int_0^t f(t-\tau)g(\tau)\, d\tau$ | $F(s)G(s)$ |
| $f(t) = f(t+p)$ | $\dfrac{1}{1-e^{-sp}} \displaystyle\int_0^p f(t)e^{-st}\, dt$ |

All $f(t)$ are defined for $t \geq 0$.

1. The *Laplace* Transform of $\cosh 2t - \sinh 2t$ is

   $(a)\ \dfrac{2s-2}{(s^2-1)^2} \quad (b)\ \dfrac{1}{s-2} \quad (c)\ \dfrac{1}{s} \quad (d)\ \dfrac{1}{s+2} \quad (e)\ \dfrac{s-2}{(s^2-4)^2}$

2. The *Laplace* Transform of $e^{3t}(t+1)$ is

   $(a)\ \dfrac{s-2}{(s-3)^2} \quad (b)\ \dfrac{2s+1}{s^2(s-3)} \quad (c)\ \dfrac{3s^2-5s-3}{s^2(s-3)} \quad (d)\ \dfrac{2s+1}{s^2(s+3)} \quad (e)\ \dfrac{s+4}{(s+3)^2}$

3. The *Laplace* Transform of $f(t) = \cos(t-3)u_3(t)$ is

   $(a)\ \dfrac{3}{s^2+9}e^{-3s} \quad (b)\ \dfrac{e^{-3}}{s(s^2+1)} \quad (c)\ \dfrac{s}{s^2+1}e^{-3s} \quad (d)\ \dfrac{s}{s^2+9}e^{-3s} \quad (e)\ \dfrac{3}{s^2+1}e^{-s}$

4. The inverse *Laplace* transform of $\frac{s}{s^2-2s+1}$ is

   $(a)\ (1-t)e^{-t} \quad (b)\ te^t \quad (c)\ e^{-t}\cos t \quad (d)\ e^t \cos t \quad (e)\ (1+t)e^t$

5. The inverse *Laplace* transform of $\frac{s-2}{s^2+5s+6}$ is

   $(a)\ e^{-3t} \quad (b)\ -\dfrac{1}{7}e^t + \dfrac{8}{7}e^{-6t} \quad (c)\ 2e^{-2t} - e^{3t} \quad (d)\ 5e^{-3t} - 4e^{-2t} \quad (e)\ e^{3t}$

6. The convolution of $t$ with $t^2$ (also denoted by $t * t^2$ ) is given by

   $(a)\ \dfrac{t^3}{6} \quad (b)\ t^3 \quad (c)\ \dfrac{t^4}{4} \quad (d)\ \dfrac{t^4}{6} \quad (e)\ \dfrac{t^4}{12}$

7. The period of $\sin\left(\frac{\pi x}{2}\right)$ is

   $(a)\ 1 \quad (b)\ 2 \quad (c)\ 2\pi \quad (d)\ 4 \quad (e)\ 4\pi$

8. The functions $f(x) = 1 - x^4$ and $g(x) = x^2 \sin x$ defined on $-1 < x < 1$ have the property that
   (a) both are even (b) both are odd (c) $f$ is odd and $g$ is even
   (d) $f$ is even and $g$ is odd (e) neither is even nor odd

9. The function $f(x) = \begin{cases} -1, & \text{if } -1 < x < 0 \\ 1, & \text{if } 0 < x < 1 \end{cases}$ is periodic with period 2.
   It has a *Fourier* Series $\sum_{n=1}^{\infty} b_n \sin(n\pi x)$ where $b_n$ is given by

   $(a)\ \dfrac{2}{n\pi}(-1)^n \quad (b)\ \dfrac{4}{n\pi}(-1)^n \quad (c)\ \dfrac{2}{n\pi}(1-\cos(n\pi)) \quad (d)\ \dfrac{2}{n} \quad (e)\ \dfrac{4}{n}$

10. The coefficient $a_0$ in the *Fourier* Series for the periodic function $f(x) = x\cos x$ if $-\pi < x < \pi$ with period $2\pi$ has the value

    $(a)\ -2 \quad (b)\ -\dfrac{4}{\pi} \quad (c)\ 0 \quad (d)\ \dfrac{2}{3} \quad (e)\ \dfrac{2}{3}\pi$