

UNIVERSITY *of* LIMERICK  
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering  
Department of Mathematics & Statistics

**MID TERM ASSESSMENT PAPER**

MODULE CODE: MA4003

SEMESTER: Autumn 2009/10

MODULE TITLE: Engineering Mathematics 3 DURATION OF EXAMINATION: 45 minutes

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 20 %

**INSTRUCTIONS TO CANDIDATES: Answer all questions. All questions carry equal marks.  
Use the Answer Sheet below.**

ANSWER SHEET

STUDENT'S NAME:

STUDENT'S ID NUMBER:

For each question, place an "X" in the box of your choice.

Question	a	b	c	d	e	Do not write in this column
1		X				
2	X					
3			X			
4					X	
5		X				
6					X	
7				X		
8			X			
9	X					
10				X		

### Table of Laplace Transforms

$f(t), t \geq 0$	$F(s) = \mathcal{L}[f(t)]$
1	$\frac{1}{s}$
$t$	$\frac{1}{s^2}$
$t^n$	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
$\frac{a}{a-b}e^{at} - \frac{b}{a-b}e^{bt}$	$\frac{s}{(s-a)(s-b)}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\int_0^t f(\tau) d\tau$	$\frac{1}{s}F(s)$
$e^{at}f(t)$	$F(s-a)$
Heaviside $u_a(t)$	$\frac{e^{-as}}{s}$
$f(t-a)u_a(t)$	$e^{-as}F(s)$
Ramp $R(t-a)$	$\frac{e^{-as}}{s^2}$
$tf(t)$	$-F'(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma) d\sigma$
$(f * g)(t) \equiv \int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
$f(t) = f(t+p)$	$\frac{1}{1 - e^{-sp}} \int_0^p f(t)e^{-st} dt$

All  $f(t)$  are defined for  $t \geq 0$ .

1. The Laplace Transform of  $t - \sinh t$  is

(a)  $\frac{-s^3 + s^2 - 1}{s^2(s^2 - 1)}$  (b)  $\frac{-1}{s^2(s^2 - 1)}$  (c)  $\frac{s^2 - s - 1}{s(s^2 - 1)}$  (d)  $\frac{1}{s^2(s^2 + 1)}$  (e)  $\frac{s^2 - s + 1}{s(s^2 + 1)}$

2. The Laplace Transform of  $e^{3t}(t + 2)$  is

(a)  $\frac{2s - 5}{(s - 3)^2}$  (b)  $\frac{2s + 1}{s^2(s - 3)}$  (c)  $\frac{3s^2 - 5s - 3}{s^2(s - 3)}$  (d)  $\frac{2s + 1}{s^2(s + 3)}$  (e)  $\frac{2s + 7}{(s + 3)^2}$

3. The Laplace Transform of  $f(t) = \cos(t - 2)u_2(t)$  is

(a)  $\frac{2}{s^2 + 4}e^{-2s}$  (b)  $\frac{e^{-2}}{s(s^2 + 1)}$  (c)  $\frac{s}{s^2 + 1}e^{-2s}$  (d)  $\frac{s}{s^2 + 4}e^{-2s}$  (e)  $\frac{2}{s^2 + 1}e^{-s}$

4. The inverse Laplace transform of  $\frac{s}{s^2 - 2s + 1}$  is

(a)  $(1 - t)e^{-t}$  (b)  $te^t$  (c)  $e^{-t} \cos t$  (d)  $e^t \cos t$  (e)  $(1 + t)e^t$

5. The inverse Laplace transform of  $\frac{s+2}{s^2+5s-6}$  is

(a)  $e^{-3t}$  (b)  $\frac{3}{7}e^t + \frac{4}{7}e^{-6t}$  (c)  $2e^{-2t} - e^{3t}$  (d)  $-\frac{1}{7}e^{-t} + \frac{4}{7}e^{6t}$  (e)  $e^{3t}$

6. The convolution of  $t$  with  $e^{-t}$  (also denoted by  $t * e^{-t}$ ) is given by

(a)  $1 - t - e^{-t}$  (b)  $(1 - t)e^{-t} - 1$  (c)  $-te^{-t}$  (d)  $te^{-t}$  (e)  $-1 + t + e^{-t}$

7. The period of  $\sin\left(\frac{\pi x}{3}\right)$  is

(a)  $\frac{3}{2}$  (b) 3 (c)  $3\pi$  (d) 6 (e)  $6\pi$

8. The functions  $f(x) = x + x^3$  and  $g(x) = x^2 \cos x$  defined on  $-1 < x < 1$  have the property that

- (a) both are even (b) both are odd (c)  $f$  is odd and  $g$  is even  
(d)  $f$  is even and  $g$  is odd (e) neither is even nor odd

9. The function  $f(x) = -x$  for  $-\pi < x < \pi$  is periodic with period  $2\pi$ . It has a Fourier Series  $\sum_{n=1}^{\infty} b_n \sin(nx)$  where  $b_n$  is given by

(a)  $\frac{2}{n}(-1)^n$  (b)  $\frac{1}{n}(-1)^n$  (c) 0 (d)  $\frac{1}{n}$  (e)  $\frac{2}{n}$

10. The coefficient  $a_0$  in the Fourier Series for the periodic function  $f(x) = x^2$  if  $-1 < x < 1$  with period 2 has the value

(a) -1 (b) 0 (c)  $\frac{1}{3}$  (d)  $\frac{2}{3}$  (e)  $\frac{2}{3}\pi^2$