UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2005/06

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**

1. (a) Sketch the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 5 \\ e^{t-5}, & \text{if } t \geq 5 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin 3t, & \text{if } 0 \leq t < 2\pi \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 2t, & \text{if } 0 \leq t < 1 \\ 4 - 2t, & \text{if } 1 \leq t < 2 \\ 0, & \text{otherwise} \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$$(i) \quad \frac{s+2}{s^2+4s+3} \qquad (ii) \quad \frac{e^{-s}}{s^2+4} \qquad (iii) \quad \ln \frac{s}{s-1}$$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 4y = f(t), \qquad y(0) = 0, \; \frac{dy}{dt}(0) = 0$$

where

$$f(t) = \begin{cases} t, & \text{if } 0 \leq t \leq 1 \\ 1, & \text{otherwise} \end{cases}$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = t + \int_0^t \sin(t-u)y(u)du$$

3. (a) Find the *Fourier* series of the periodic function 6

$$f(x) = \begin{cases} -1, & \text{if } -1 < x \leq 0 \\ 1, & \text{if } 0 < x \leq 1 \end{cases} \qquad f(x+2) = f(x)$$

(b) Hence evaluate the alternating series 4

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(c) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 6

$$\frac{d^2y}{dx^2} - y = f(x)$$

where $f(x)$ is as in part (a).

4. (a) Let P_2 be the space of polynomials of degree at most two, i.e. $P_2 = \{p(x) : p(x) = a_0 + a_1x + a_2x^2\}$ where $a_0, \; a_1$ and a_2 are real numbers. Determine whether the set of polynomials

$$\{1 + x, \; 2 - 2x + x^2, \; 1 + 3x + x^2\}$$

forms a basis for P_2 8

(b) Find bases for the row space and column space, and hence the rank, of the matrix 8

$$\begin{pmatrix} 1 & -2 & 0 \\ 2 & 5 & 18 \\ 3 & -3 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

5. (a) Let P_2 be the space of polynomials of degree at most two. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x,\ x^2\}$ for P_2 to an orthonormal one defined by the inner product

$$< p, q >= \sum_{i=1}^{4} p(x_i)q(x_i)$$

where $x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 3.$ 10

(b) Let f be a function on $[0, 3]$, taking the value $f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4$ as given in the following table

x_i	$f(x_i)$
0	3
1	2
2	5
3	4

Find the least squares approximation to f in P_2 using the inner product defined in part (a). 6

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{pmatrix}$$

Is A diagonalisable over the reals ? If so, find the matrix P which diagonalises it. Use the diagonalisation to calculate A^4. 8

7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 4

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 10 & -3 \\ 1 & -3 & 10 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 4

$$b = \begin{pmatrix} 0 \\ 3 \\ 25 \end{pmatrix}$$

(b) Using the factorisation $A = LL^T$ found in part (a), solve the systems of equations $Aw^{(i)} = y^{(i)},\ i = 1, 2$ where $y^{(1)} = (1, 0, 0)^T$ and $y^{(2)} = (0, 1, 0)^T$ to obtain an estimate for $||A^{-1}||$ and hence estimate the condition number of the matrix A. 8