UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2011/12

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. T. Myers

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**
A table of *Laplace* transforms is attached.

1. (a) Sketch the graphs of the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 2 \\ t-2, & \text{if } t \geq 2 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin \pi t, & \text{if } 0 \leq t < 3 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 0, & \text{if } t < 0 \\ 5, & \text{if } 0 \leq t < 1 \\ 4, & \text{if } 1 \leq t < 2 \\ f(t-2), & \text{if } t \geq 2 \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$(i)\ \dfrac{2s-2}{s^2-s-6} \qquad (ii)\ \dfrac{e^{-s}}{s^2+4s+4} \qquad (iii)\ \tan^{-1}(s+2)$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 4y = 5e^{-t}, \qquad y(0) = 1, \ \frac{dy}{dt}(0) = 0$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = \sin 3t - 2\int_0^t \cos 3(t-u)y(u)du$$

3. (a) Sketch the graph of the following periodic function and find its *Fourier* series 6

$$f(x) = \begin{cases} 1, & \text{if } -\pi < x \leq -\frac{\pi}{2} \\ 0, & \text{if } -\frac{\pi}{2} < x \leq \frac{\pi}{2} \\ 1, & \text{if } \frac{\pi}{2} < x \leq \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

(b) Hence evaluate the series 4

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(c) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 6

$$\frac{d^2y}{dx^2} + 2y = f(x)$$

where $f(x)$ is as in part (a).

4. Let P_2 be the space of polynomials of degree at most two, i.e.
$P_2 = \{p(x) : p(x) = a_0 + a_1 x + a_2 x^2\}$ where a_0, a_1 and a_2 are real numbers.

   (a) Determine whether the set of polynomials

$$Q = \{1 + 2x,\ 2x + x^2,\ 4 + 2x - 3x^2\}$$

   forms a basis for P_2 6

   (b) Define span (Q). What is the dimension of span (Q) ? 4

   (c) Is $-3 + 3x^2 \in$ span (Q) ?
   For what value of a_1 is $1 + a_1 x + x^2 \in$ span (Q) ? 6

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x\}$ for P_1 to an orthonormal one defined by the inner product

$$< p, q >= \sum_{i=1}^{5} p(x_i) q(x_i)$$

   where $x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 3,\ x_5 = 4.$ 8

   (b) Let f be a function on $[0, 4]$, taking the value $y_i = f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4, 5$ as given in the following table

x_i	y_i
0	4
1	3
2	1
3	0
4	-1

   Find the least squares approximation to f in P_1 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{pmatrix}$$

Show that A is diagonalisable over the reals and find the matrix P which diagonalises it. Hence or otherwise solve the system of differential equations

$$\frac{d}{dt} X = AX, \qquad X(0) = (1, 1, 0)^T$$

8

7. (a) Find the LU decomposition ($A = LU$) with a unit lower triangular matrix of the matrix 5

$$A = \begin{pmatrix} 1 & -3 & 1 \\ 2 & -5 & 4 \\ 2 & -2 & 11 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 3

$$b = \begin{pmatrix} 0 \\ 5 \\ 22 \end{pmatrix}$$

(b) Define the condition number of a square matrix. Of what is it a measure ? 2

For the matrix A of part (a), use the LU decomposition or otherwise to calculate its inverse, and then calculate its condition number using the maximum row-sum norm. 6