UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2010/11

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Dr. T. Myers

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**
A table of *Laplace* transforms is attached.

1. (a) Sketch the graphs of the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 1 \\ e^{-(t-1)} \sin 2(t-1), & \text{if } t \geq 1 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \cos 3\pi t, & \text{if } 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 0, & \text{if } t < 0 \\ 2, & \text{if } 0 \leq t < 1 \\ 1, & \text{if } 1 \leq t < 2 \\ f(t-2), & \text{if } t \geq 2 \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$$(i) \ \frac{2s-4}{s^2-5s-6} \qquad (ii) \ \frac{e^{-2s}}{s^2+4s} \qquad (iii) \ \ln(s^2+4)$$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 1, \qquad y(0) = 0, \ \frac{dy}{dt}(0) = 1$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = e^{-2t} + 2\int_0^t e^{-2(t-u)} y(u)\,du$$

3. (a) Sketch the graph of the following periodic function and find its *Fourier* series 7

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & \text{if } -\pi < x \leq 0 \\ -x + \frac{\pi}{2}, & \text{if } 0 < x \leq \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

(b) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 9

$$\frac{d^2y}{dx^2} + 2y = f(x)$$

where $f(x)$ is as in part (a).

4. (a) Let P_2 be the space of polynomials of degree at most two, i.e. $P_2 = \{p(x) : p(x) = a_0 + a_1 x + a_2 x^2\}$ where $a_0,\ a_1$ and a_2 are real numbers. Determine whether the set of polynomials

$$\{1 - 2x,\ 2 + x^2,\ 4x + x^2\}$$

forms a basis for P_2 8

(b) Determine whether the following set of vectors in $\mathbf{R}^4$ is linearly independent. 8

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ -3 \\ -3 \end{pmatrix} \right\}$$

5. (a) Let P_2 be the space of polynomials of degree at most two. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x,\ x^2\}$ for P_2 to an orthonormal one defined by the inner product

$$< p, q >= \sum_{i=1}^{4} p(x_i)q(x_i)$$

where $x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 3.$ 8

(b) Let f be a function on $[0, 3]$, taking the value $y_i = f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4$ as given in the following table

x_i	y_i
0	4
1	1
2	3
3	6

Find the least squares approximation to f in P_2 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

Show that A is diagonalisable over the reals and find the matrix P which diagonalises it. Hence or otherwise calculate A^4. 8

7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 5

$$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 20 & -2 \\ 1 & -2 & 10 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 3

$$b = \begin{pmatrix} -1 \\ 18 \\ -10 \end{pmatrix}$$

(b) By considering the eigenvalues of the iteration matrix, or otherwise, show that the Jacobi method is guaranteed to converge for the system of linear equations 5

$$\begin{pmatrix} 1 & 1 & -2 \\ 2 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \\ 3 \end{pmatrix}$$

Use three (3) iterations of the method, with initial guess $x_1(0) = 0,\ x_2(0) = 0,\ x_3(0) = 0$, to obtain a numerical solution of the system. 3