



UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2008/09

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

**INSTRUCTIONS TO CANDIDATES: Full marks for correct answers to 5 questions.
A table of *Laplace* transforms is attached.**

1. (a) Sketch the following functions and find their *Laplace* transforms:

$$(i) f(t) = \begin{cases} 0, & \text{if } t < 2 \\ (t-2)e^{-(t-2)}, & \text{if } t \geq 2 \end{cases} \quad 2$$

$$(ii) f(t) = \begin{cases} \cos \pi t, & \text{if } 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases} \quad 3$$

$$(iii) f(t) = \begin{cases} 0, & \text{if } t < 0 \\ 2, & \text{if } 0 \leq t < 1 \\ 1, & \text{if } 1 \leq t < 2 \\ f(t-2), & \text{if } t \geq 2 \end{cases} \quad 3$$

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$$(i) \frac{s+2}{s^2-5s+6} \quad (ii) \frac{e^{-5s}}{s^2-s} \quad (iii) \tan^{-1} s$$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \cos 2t + \frac{1}{2}\sin 2t, \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 0$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = 1 + t - 4 \int_0^t (t-u)y(u)du$$

3. (a) Find the *Fourier* series of the periodic function 5

$$f(x) = \begin{cases} 0, & \text{if } -2 < x \leq -1 \\ 1, & \text{if } -1 < x \leq 1 \\ 0, & \text{if } 1 < x \leq 2 \end{cases} \quad f(x+4) = f(x)$$

(b) Hence evaluate the series 5

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

(c) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 6

$$\frac{d^2y}{dx^2} + y = f(x)$$

where $f(x)$ is as in part (a).

4. (a) Let P_3 be the space of polynomials of degree at most three, i.e. $P_3 = \{p(x) : p(x) = a_0 + a_1x + a_2x^2 + a_3x^3\}$ where a_0, a_1, a_2 and a_3 are real numbers. Determine whether the set of polynomials

$$\{1 - 2x, 2 + x^2, -1 + 4x + 3x^2, -x + x^3\}$$

forms a basis for P_3

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- (b) Find bases for the row space and column space, and hence the rank, of the matrix

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$$\begin{pmatrix} 1 & -2 & 0 \\ 2 & 5 & 18 \\ 3 & -3 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1, x\}$ for P_1 to an orthonormal one defined by the inner product

$$\langle p, q \rangle = \sum_{i=1}^5 p(x_i)q(x_i)$$

where $x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2, x_5 = 3$.

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- (b) Let f be a function on $[-1, 3]$, taking the value $y_i = f(x_i)$ at $x = x_i, i = 1, 2, 3, 4, 5$ as given in the following table

x_i	y_i
-1	6
0	3
1	2
2	0
3	-1

Find the least squares approximation to f in P_1 using the inner product defined in part (a).

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6. Find the eigenvalues and corresponding eigenspaces of the matrix

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$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{pmatrix}$$

Show that A is diagonalisable over the reals and find the matrix P which diagonalises it. Hence or otherwise calculate A^4 .

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7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix

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$$A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 5 & -4 \\ 2 & -4 & 6 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where

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$$b = \begin{pmatrix} -1 \\ 7 \\ -6 \end{pmatrix}$$

- (b) By considering the eigenvalues of the iteration matrix, or otherwise, show that the Jacobi method is guaranteed to converge for the system of linear equations

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$$\begin{pmatrix} 1 & 1 & -2/5 \\ 2/5 & 1 & 2/5 \\ 2/5 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -2 \end{pmatrix}$$

Use three (3) iterations of the method, with initial guess $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, to obtain a numerical solution of the system.

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