

Homework: Fibonacci Numbers +

1. Consider the Fibonacci numbers:

$$1, 1, 2, 3, 5, 8, \dots,$$

(a) What is F_{16} ?

(b) Prove that

$$F_1 + F_2 + \dots + F_n = F_{n+2} - 1.$$

One method for doing this and also for the following part is to use mathematical induction, i.e. (i) prove that it is true for $n=1$ and (ii) prove that if it is true for n , then it is true for $n + 1$.

(c) Prove that

$$F_1^2 + F_2^2 + \dots + F_n^2 = F_n F_{n+1}.$$

Hint:

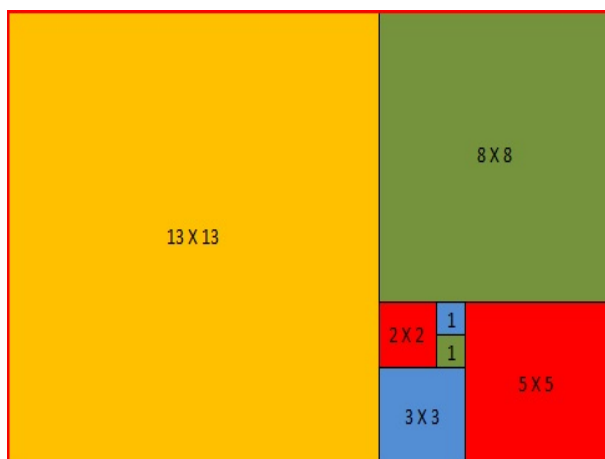


Figure 1: Every non-square rectangle can be decomposed as a sum of squares

(d) The Lucas numbers are defined by the recurrence relation

$$L_n = L_{n-1} + L_{n-2}, \quad n \geq 2$$

together with the two specified initial terms: $L_1 = 2$ and $L_2 = 1$, i.e. they obey the same recurrence as the Fibonacci numbers, but with different initial terms. What is L_8 ?

(e) Define $B_n = L_n - F_n$. Find the recurrence satisfied by B_n (Your answer should only involve terms of the B_n sequence). What are its initial values? By inspection of the B_n sequence, can you find a relationship between it and the Fibonacci sequence?