Minimax Game Solution

An alternative to "solving " matrix games using the concept of *Nash* Equilibrium is the Minimax approach. It was originally devised for 2-player "Zero Sum" or "Constant Sum" games where what one player gains the other player loses. Each player attempts to maximise hir payoff assuming that hir opponent is attempting to minimise it.

Consider the constant sum game

		Player 2	
		Left	Right
Player 1	Up	(5,3)	(2,6)
	Down	(3,5)	(4,4)

As usual we'll use the notation $R_1\langle s_1, s_2\rangle$ and $R_2\langle s_1, s_2\rangle$ to stand for the payoffs to Player 1 and 2 respectively when Player 1 plays strategy s_1 and Player 2 strategy s_2. Again let p be the probability that Player 1 plays Up, and q the probability that Player 2 plays Left. Then the payoff to player 1 is

$$R_1\langle p\text{Up}+(1-p)\text{Down}, q\text{Left}+(1-q)\text{Right}\rangle = 5pq+2p(1-q)+3(1-p)q+4(1-p)(1-q) = 4pq-2p-q+4$$

while the payoff to player 2 is

$$R_2\langle p\text{Up}+(1-p)\text{Down}, q\text{Left}+(1-q)\text{Right}\rangle = 3pq+6p(1-q)+5(1-p)q+4(1-p)(1-q) = -4pq+2p+q+4$$

(Note the constant sum $R_1\langle\cdot,\cdot\rangle + R_2\langle\cdot,\cdot\rangle = 8$)

Analysis

If R_1 is an increasing function of q (i.e. $\frac{\partial R_1}{\partial q} > 0$) then Player 2 minimises Player 1's payoff by choosing $q = 0$. If R_1 is an decreasing function of q (i.e. $\frac{\partial R_1}{\partial q} < 0$) then Player 2 minimises Player 1's payoff by choosing $q = 1$. If however $\frac{\partial R_1}{\partial q} = 0$ then Player 2 can't influence Player 1's payoff.
In this case $\frac{\partial R_1}{\partial q} = 4p - 1$. Thus

(i) $4p - 1 > 0 \Rightarrow q = 0$ and

$$R_1 = 4 - 2p < 4 - 2(1/4) = 3.5$$

(ii) $4p - 1 < 0 \Rightarrow q = 1$ and

$$R_1 = 3 + 2p < 3 + 2(1/4) = 3.5$$

(iii) $4p - 1 = 0$ then irrespective of q

$$R_1 = 3.5$$

Hence Player 1 should choose $p = 1/4$ as hir optimal strategy, and $v_1 = 3.5$.

A similar analysis can be done with R_2 as a function of p. Here $\frac{\partial R_2}{\partial p} = -4q + 2$. Thus

(i) $-4q + 2 > 0 \Rightarrow p = 0$ and

$$R_2 = 4 + q < 4 + 1/2 = 4.5$$

(ii) $-4q + 2 < 0 \Rightarrow p = 1$ and

$$R_2 = 6 - 3q < 6 - 3(1/2) = 4.5$$

(iii) $-4q + 2 = 0$ then irrespective of p

$$R_2 = 4.5$$

Hence Player 2 should choose $q = 1/2$ as hir optimal strategy, and $v_2 = 4.5$

The minimax strategies are thus $\langle(1/4)\text{Up} + (3/4)\text{Down}, (1/2)\text{Left} + (1/2)\text{Right}\rangle$.